Astro 102/104 Our Solar System **Spring 2008**

NAME:______________________________ Section Number:______________

Homework 12: Solar System Origin, Sun, Extrasolar Planets

Due: in your section on **the week of April 28th**. Be neat and concise, show your work, and remember units. An answer without the correct units is wrong.

Suggested reading: Chapters 8, 13, 14, and lecture notes 33-36.

1 Solar Nebula Theory

a. [1 point] What was the *frost line* in the solar nebula, and where was it located?

The frost line is the distance from a star beyond which the nebula temperature is low enough for hydrogen compounds (especially water) to condense into ices. Inside the frost line, the high temperatures force these compounds to remain in a gaseous state. We think the frost line in our solar nebula was located between the present-day orbits of Mars and Jupiter, i.e. between 2 and 5 AU.

b. [2 points] How does the frost line explain the categorical differences in composition and mass between the terrestrial and jovian planets in our solar system?

The terrestrial planets formed inside the frost line, through accretion of solid particles composed of metals and silicates (rocks). The jovian planets formed outside the frost line, initially from solid particles composed of metals, silicates, and hydrogen compounds (e.g., water, ammonia, and methane). Because hydrogen compounds were much more abundant than silicates and metals in the solar nebula, the jovian planet cores grew several times more massive than the terrestrial planets, until they were massive enough to attract (gravitationally) the even more abundant hydrogen and helium gases distributed throughout the nebula. Thus, the jovian planets were able to incorporate all types of matter present in the solar nebula, whereas the terrestrial planets formed from just the small fraction of nebular material that was solid inside the frost line. This resulted in the smaller size and different composition of the terrestrial planets.

c. [2 points] Given your answer to part (b), why was it surprising to find “hot Jupiter” extrasolar planets orbiting very close to their stars? How can we explain their small orbital semi-major axes?

It was surprising to find planets as massive as Jupiter (or larger!) orbiting at ~0.1 AU or less from their stars because these distances must have been inside the frost line, where ices were not stable and thus it would seem impossible for planet cores to grow massive enough to attract hydrogen/helium envelopes. But without attracting hydrogen and helium, we cannot imagine how a planet could become as massive as Jupiter. Thus, astronomers think that the “hot Jupiters” did form outside their respective frost lines, but then migrated inward to their present locations relatively late in the process of planet formation.

d. [2 points] Aside from the differences between terrestrial and jovian planets, list two other observational constraints for solar system formation. **Explain** how each is accounted for by the solar nebula theory.

All eight planets orbit the Sun in nearly the same plane (the ecliptic), and in the same direction. Most—though not all—of the planets (and the Sun itself) rotate in this same direction, with fairly small obliquities. These are natural results of the solar nebula theory, which stipulates that the planets formed from particles orbiting in a disk. Also, the oldest materials in the solar system, which are found in meteorites, all have ages of roughly 4.56 billion years, while the oldest rocks on the Moon's surface are approximately 4.4 billion years old. This implies that the Moon formed in less than 200 million years, as predicted by the solar nebula theory (computer models show that Moon- and even Earth-sized objects should form in only 10-100 million years). Finally, the existence of asteroids and trans-Neptunian objects is readily explained by the solar nebula theory: they are leftover planetesimals that were never incorporated into planets.

2 Fusion in the Sun

a. [2 points] When four protons fuse to form a helium atom in the Sun (with multiple intermediate steps), 0.7% of their mass is converted to energy according to Einstein's mass-energy equation. The mass of a proton is 1.67×10^{-27} kg. Calculate the energy released by a single fusion reaction.

The mass converted into energy is 0.7% of the mass of four protons:

$m = 0.007 * 4 * 1.67\text{x}10^{-27}\ \text{kg} = 4.68\text{x}10^{-29}\ \text{kg}$

The energy released is therefore:

$E = mc^2 = (4.68\text{x}10^{-29}\ \text{kg}) * (3.0\text{x}10^{8}\ \text{m/s})^2 = \mathbf{4.21x10^{-12}\ J}$

b. [5 points] Your textbook (p. 501) says that the above reaction occurs roughly 10^{38} times each second. At this rate, given that the Sun's total mass is roughly $2\text{x}10^{30}$ kg, and 70% of this is hydrogen, how many years would it take to convert all of the Sun's hydrogen atoms into helium? Will the Sun have fused all of its hydrogen by the time it becomes a red giant in ~5 billion years?

The total mass of hydrogen in the Sun is $0.7 * 2\text{x}10^{30}\ \text{kg} = 1.4\text{x}10^{30}\ \text{kg}$.

The mass of hydrogen fused in each reaction is $4 * (1.67\text{x}10^{-27}\ \text{kg}) = 6.68\text{x}10^{-27}\ \text{kg}$.

Thus, the number of reactions needed to fuse all the Sun's hydrogen is:

$(1.4\text{x}10^{30}\ \text{kg})/(6.68\text{x}10^{-27}\ \text{kg}) = 2.1\text{x}10^{56}$

To complete all these reactions at a rate of 10^{38} *per second would require* $2.1\text{x}10^{56}/(10^{38}\ \text{s}^{-1}) = 2.1\text{x}10^{18}$ *s. Converting to years, this is* $(2.1\text{x}10^{18}\ \text{s}) * (1\ \text{year}/3.15\text{x}10^{7}\ \text{s}) = \mathbf{6.65x10^{10}}$ ***years, i.e. over 66 billion years.*** *Clearly, the Sun will* ***not*** *fuse all its hydrogen by the end of its lifetime; rather, it will fuse only the hydrogen in its core, where temperatures are high enough for nuclear fusion to occur.*

3 Extrasolar Planets

a. [2 points] What technique has been used to discover most of the known extrasolar planets? What can this technique tell us about each planet it discovers?

The Doppler Shift (a.k.a. Radial Velocity) method has been used to discover most of the extrasolar planets found to date. This method provides an estimate of a planet's mass, although the exact mass cannot be derived without knowing the inclination of the planet's orbit to our line of sight. It also provides the planet's orbital period, which can be used to infer its semimajor axis (using Kepler's 3rd law!) given an estimate of its star's mass. Measurements of the rate at which the Doppler shift changes throughout the planet's orbit can even be used to compute its eccentricity.

b. [2 points] Search online for information about the transiting planet HD 189733b. What molecules have recently been discovered in its atmosphere? Do these molecules occur in the atmospheres of jovian planets in our solar system?

Water (H_2O) and methane (CH_4) have recently been discovered in the atmosphere of HD 189733b (Swain et al., 2008; Nature 452, 329). This detection of methane marks the first observation of an organic molecule in an extrasolar planet's atmosphere! These molecules also occur in the atmospheres of all the jovian planets in our solar system, although only at Uranus and Neptune does the temperature get low enough for methane to condense and form clouds, whereas only at Jupiter and Saturn are temperatures high enough in the upper atmosphere for us to see all the way down to the level of water clouds. The upper atmosphere of HD 189733b is so hot (>1000 K!) that these volatiles will not form clouds, but there may be clouds of the rock-forming silicate mineral $MgSiO_3$! (Lecavelier Des Etangs et al., 2008; A&A 481, L83)

c. [2 points] Most known extrasolar planets are more massive than the most massive planet in our own solar system (Jupiter), and orbit closer to their stars than Jupiter does to the Sun. Does this mean our solar system is unusual? Why or why not?

No, we cannot necessarily infer that our solar system is unusual (although we cannot yet rule it out, either), because our observations so far probably represent a biased sample. The Doppler Shift method—and most other methods—for finding extrasolar planets are most sensitive to massive planets orbiting close to their stars. Thus we should not yet conclude that low-mass planets and distant "cold Jupiters" like ours are rare. We won't know for sure until astronomers are able to detect these types of planets, and determine if they are truly absent from other planetary systems.