

Name: Solution

Section: _____

Homework 2: Newton's Laws, Kepler's laws, Seasons, and a review of the Celestial Sphere

Due: In your section the week of **February 4th**. Be neat and concise, show your work, and remember units. An answer without the correct units is wrong.

1. (2 points) **Mass and weight.** The Mars Exploration Rover Spirit has a mass of 185 kg. The SI unit of force is the Newton, written as N. $1 \text{ N} = 1 \text{ kg m/s}^2$. Look at Appendix E (Planetary Data) in your book for help with part b).

a) What did Spirit weigh when she was on Earth (after she was fully assembled)?

$$\begin{aligned} W &= mg \\ &= (185 \text{ kg}) * (9.8 \text{ m/s}^2) \\ &= 1813 \text{ N (or } 1800 \text{ N if you're counting significant figures)} \end{aligned}$$

b) What does she weigh on Mars?

According to Table E.1, the surface gravity on Mars is 0.38 times the surface gravity on Earth.

$$\begin{aligned} W &= 0.38 * (1813 \text{ N}) \\ &= 689 \text{ N (or } 690 \text{ N if you're counting significant figures)} \end{aligned}$$

2. (2 points) **Newton's laws.** Given a constant force F:

a) What happens to the acceleration if you double the mass?

$$\text{The acceleration halves: } F = ma = (2m) * (1/2 a)$$

b) What happens to the acceleration if you halve the mass?

$$\text{The acceleration doubles: } F = ma = (1/2 m) * (2a)$$

3. (1 point) **Seasons.** Why is it warmer in summer and colder in winter?

Because of the obliquity (tilt) of the Earth, sunlight falls more directly on the Northern Hemisphere during part of the year (Northern summer) and less directly six months later (Northern winter). The greater amount of sunlight in summer heats the air, land and ocean more than the lesser amount of sunlight of winter.

4. (3 points) **Astronomical time.**

a) What is a sidereal day? How is it different from a solar day?

A sidereal day is the time it takes for a particular star to return to the same place in the sky. The Earth takes 24 hours (one solar day) to rotate once on its axis. Also, the Earth is revolving around the Sun in the same direction as its rotation. This added motion means that a star will lead its previous location by about 4 minutes.

b) Draw a sketch of a sidereal day. Indicate the Earth, the Sun, and far away (“fixed”) stars.

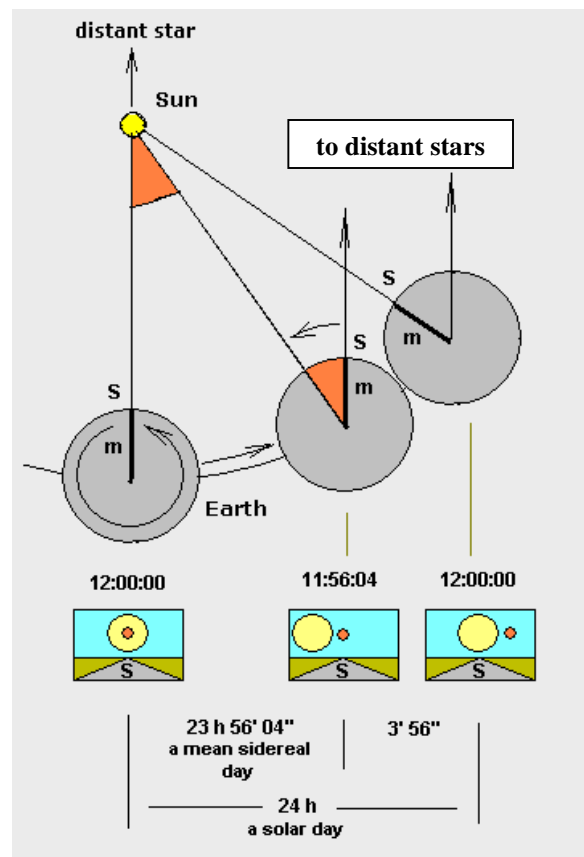
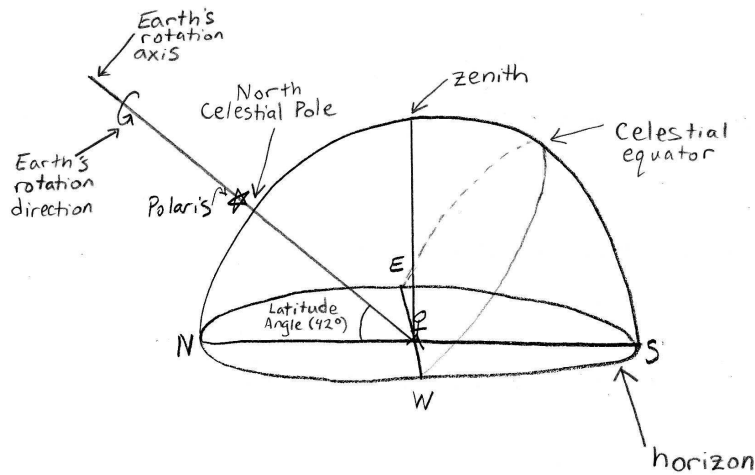


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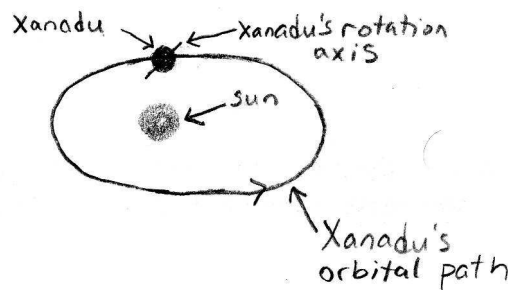
4. (6 points) **Celestial Sphere.** Label this celestial sphere, including the latitude angle (assume the person is in Ithaca), zenith, horizon, north celestial pole, Earth's rotation axis (including the direction), and celestial equator.



6. (6 points) **Bringing it all together.** In this question you will calculate the size of an imaginary planet, step-by-step, using the method that Eratosthenes used to calculate the size of the Earth. Then you will apply some of Newton's and Kepler's Laws to this planet-sun system.

Scenario: You're an astronomy professor at the University of Utopia, in the northern hemisphere of the planet Xanadu. You have two grad students, Alice and Bob. You and Alice are attending a conference in Arcadia, a city 400km north of Utopia, on the spring equinox. You leave Bob at home to teach your classes in your absence.

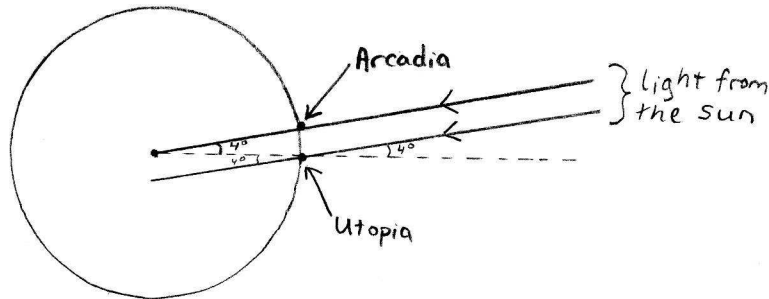
- a) **Seasons.** Draw a diagram showing where Xanadu is in relation to its sun on the equinox. Assume Xanadu's obliquity is similar to Earth's.



- b) **What the ancients knew.** You tell Alice and Bob to measure the altitude of the sun on the equinox. Alice measures it as 90° in Arcadia, while Bob gets 86° . How many degrees of latitude separate Arcadia and Utopia?

The latitude separation of the two cities is equal to the difference in the altitude of the sun: 4 degrees.

- c) Draw the equivalent of the picture from the Special Topic on p.68 of your book. Be sure to label Arcadia, Utopia, and the angles.



- d) You now have all of the information you need to figure out Xanadu's size. By comparing the latitude (degrees) and distance (km) separation of Arcadia and Utopia to the geometry of a sphere, calculate Xanadu's circumference.

The ratio of the latitude separation of the two cities to a full circle (a great circle or line of longitude of the planet) is the same as the ratio of the distance separation of the cities to the circumference of the circle:

$$\begin{aligned} (4^\circ / 360^\circ) &= (400 \text{ km} / \text{circumference}) \\ \text{circumference} &= (360^\circ / 4^\circ) * 400 \text{ km} \\ \text{circumference} &= 36,000 \text{ km} \end{aligned}$$

- e) **Kepler's laws.** If Xanadu's semimajor axis is 1.2 AU, how long is its year?

$$\begin{aligned} p^2 &= a^3 \\ p^2 &= (1.2 \text{ AU})^3 \\ p &= (1.2 \text{ AU})^{3/2} \\ p &= 1.3 \text{ years} \end{aligned}$$

- f) **Gravitation.** What is the gravitational force between Xanadu and its sun? Assume that Xanadu has the same mass as the Earth, and Xanadu's sun has the same mass as the Sun.

Use the gravity equation: $F = G M_1 M_2 / R^2$

First we have to convert 1.2 AU (the distance from Xanadu to its sun) into meters.

$$\begin{aligned} R &= (1.2 \text{ AU}) * (149.6 \times 10^6 \text{ km} / 1 \text{ AU}) * (10^3 \text{ m} / 1 \text{ km}) \\ &= 1.8 \times 10^{11} \text{ m} \end{aligned}$$

$$\begin{aligned} F &= G M_1 M_2 / R^2 \\ &= (6.67 \times 10^{-11} \text{ m}^3 / \text{kg s}^2) * (5.97 \times 10^{24} \text{ kg}) * (1.99 \times 10^{30} \text{ kg}) / (1.8 \times 10^{11} \text{ m})^2 \\ &= 2.45 \times 10^{22} \text{ N} \end{aligned}$$

($F = 2.5 \times 10^{22} \text{ N}$ if you're counting significant figures)