Problem 1:

This is the case of a single plano-convex lens. The specifications are:

Focal length	f	~ 15 cm
Diameter	D	= 10 cm
Index of refraction	n	= 2.1
Size of aperture stop	s	= 9 cm
Thickness of lens	t	= 1 cm

a) From the lensmaker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \text{ with } r_1 \to \infty \text{ , } r_2 = -(n-1)f$$

Thus, r_2 = -1.1·15 = 16.5 cm

b) Do a system plot and spot diagram with ray trace program.

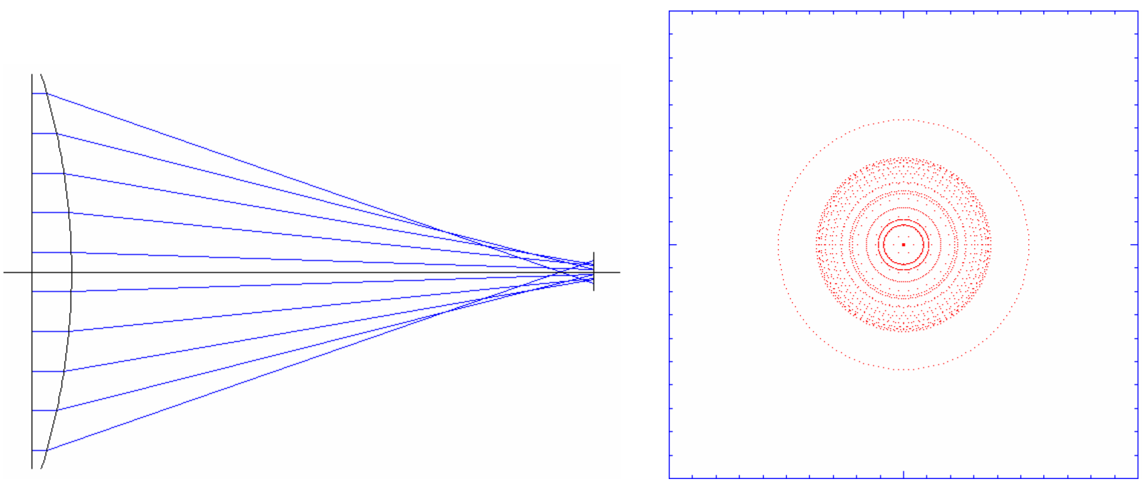

Figure 1-1: Left is system plot of the p-c lens. On the right is a spot diagram. Full scale is 1 cm. File is: pc with conic surface (f = 15).ray

c) rms (on-axis) = 0.1516 cm, rms (1 deg. off-axis) = 0.1535

d) Vary eccentricity to minimize the rms spot size.

Eccentricity = 2.10 Best case - see Figure 1-2 below.
rms (on-axis) = 0.0 cm rms (1 deg. off-axis) = 0.0192 cm

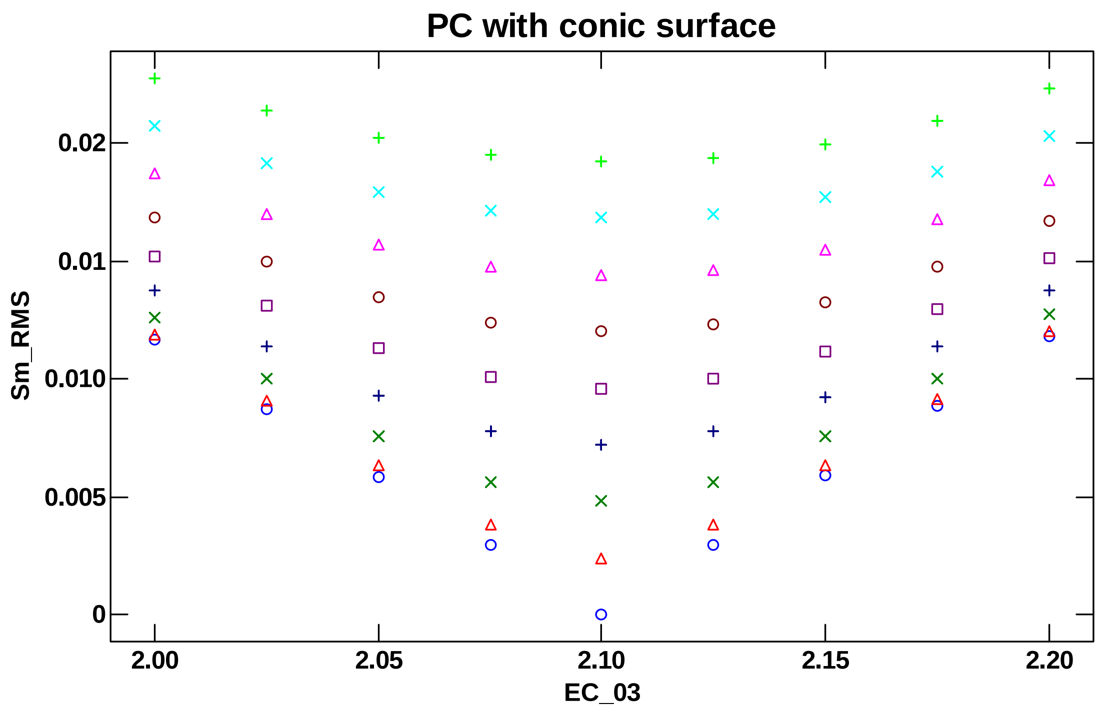

Figure 1-2: Optimization plot of smallest rms vs. eccentricity of the curved surface of the plano-convex lens. Different symbols represent different distances off-axis (from 0 to 1.0 degree). The best eccentricity is 2.1.

e) Show spot diagrams for the eccentricity optimized case.

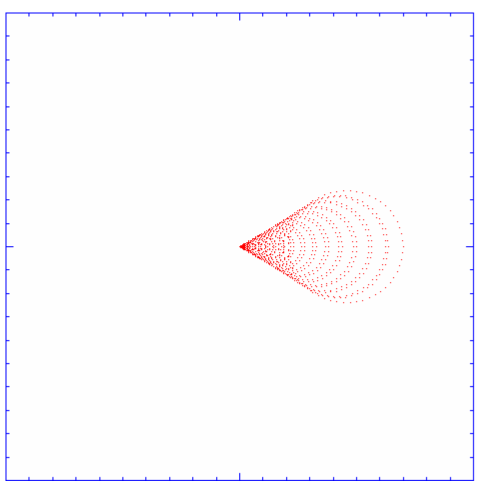

Figure 1-3: Spot diagram for p-c lens with e = 2.1. The source is one degree off-axis. The on-axis case gives a perfect image. The full scale is 0.15 cm.

f) Do the same calculations for a convex plano-lens.

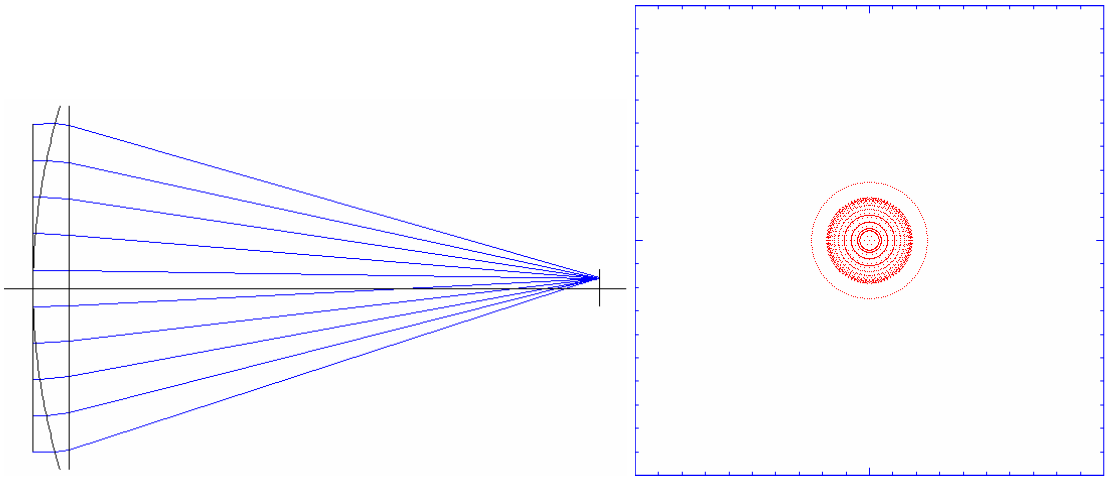

Figure 1-4: Left is system plot of the c-p lens. On the right is a spot diagram. Full scale is 0.5 cm. File is: cp with conic surface (f = 15).ray

rms (on-axis) = 0.0368 cm, rms (1 deg. off-axis) = 0.0372

Vary eccentricity to minimize the rms spot size.

Eccentricity = 1.09 Best case - see Figure 1-5 below.
rms (on-axis) = 0.000671 cm rms (1 deg. off-axis) = 0.00584 cm

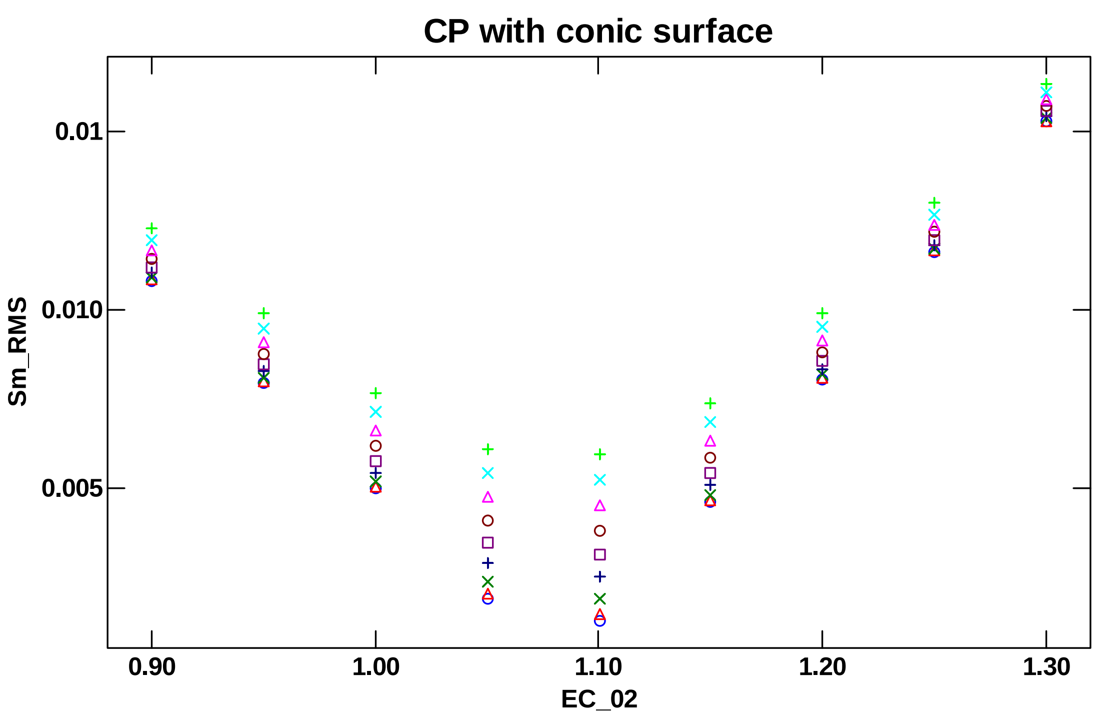

Figure 1-5: Optimization plot of smallest rms vs. eccentricity of the curved surface of

the convex-plano lens. Different symbols represent different distances off-axis (from 0 to 1.0 degree). The best eccentricity is 2.1.

Show spot diagrams for the eccentricity optimized case.

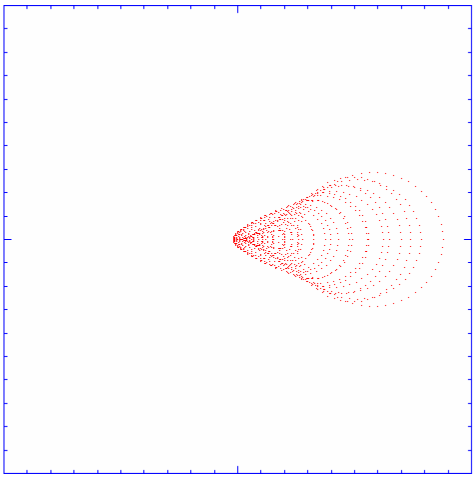

Figure 1-6: Spot diagram for c-p lens with e = 1.09. The source is one degree off-axis. The on-axis case gives a near perfect image. The full scale is 0.05 cm.

g) Which case is better?

Clearly the C-P lens is better both with and w/o optimization. Table 1a summarizes the cases below.

Table 1a: Summary of P-C/C-P comparison

Case	Eccentricity	Rms spot size	
		$\theta = 0°$ (cm)	$\theta = 1°$ (cm)
P-C	0.00	0.1516	0.1535
P-C	2.10	0.0000	0.0192
C-P	0.00	0.0368	0.0372
C-P	1.09	0.000671	0.00584

Problem 2:

A summary of results is given in Table 2a:

Table 2a: Ray-trace results for double lens system

	First Lens				Second Lens				rms spot size	
Case	r_1 (cm)	e_1	r_2 (cm)	e_2	r_1 (cm)	e_1	r_2 (cm)	e_2	$\theta = 0°$ (cm)	$\theta = 1°$ (cm)
1	0	0	-31.14	0	31.14	0	0	0	0.043614	0.045103
2	0	0	-31.14	2.41	31.14	2.584	0	0	0.000086	0.012463
3	29.586	0	114.95	0	10.101	0	15.699	0	0.003382	0.003758
4	29.586	0	114.95	0	10.101	0.2127	15.699	0.2	0.000008	0.001968
2a	0	0	-31.14	2.25	31.14	2.865	0	0	0.000195	0.012789

Cases:

1. As designed from two-lens, thin lens, and lensmakers equations.
2. Did a "hand" search starting with both eccentricities equal to 2.576.
3. Bent lenses to reduce spot size
4. Same as case 3 but optimized by varying eccentricity of surfaces of second lens

2a. Did a grid search with variation window

Details of the Solution:

For a two lens system separated by a distance *d*, the object a distance *o* from the first (leftmost) vertex and the image a distance *i* from the rightmost vertex, we have

$$i = \frac{of_1 f_2 - d(f_1 + o) f_2}{of_1 + (f_2 - d)(f_1 + o)} \xrightarrow[o \to -\infty]{} \frac{f_1 f_2 - df_2}{f_1 + f_2 - d} \xrightarrow[f_1 = f_2]{} \frac{f_1^{\,2} - df_1}{2 f_1 - d}$$

Where the first limit is for the object at minus infinity and the second limit has the two focal lengths set equal. We can solve the above equation for the focal length, f_1

$$f_1 = i + d/2 + \sqrt{(i + d/2)^2 - id}$$

The principal plane will be located at the stop because of the symmetry of the system so the focal length, *f*, of the doublet will be $i+d/2$. Putting this into the above equation yields the focal length of the individual lenses in terms of the total focal length and lens separation.

$$f_1 = f + \sqrt{f^2 - (f - d/2)d}$$

Since $d = 2$ cm and we want $f = 15$ cm, $f_1 = 29.04$ cm. For a plano-convex lens the lensmakers equation simplifies

$$r = -(n-1) f_1$$

for the curved surface and so for $n = 2.1$, we have $r = 31.91$ cm.

Case 1: If we used the back focal distance, i = 15 cm instead of f we get f_1 = 31.03 cm and r = 34.14 cm. As per the instructions in the problem, we will us these latter numbers. The results of a trace with 2000 rays are given in Table 2a and spot diagrams for on-axis and one degree off-axis are shown below. The back focal distance is 13.39 cm.

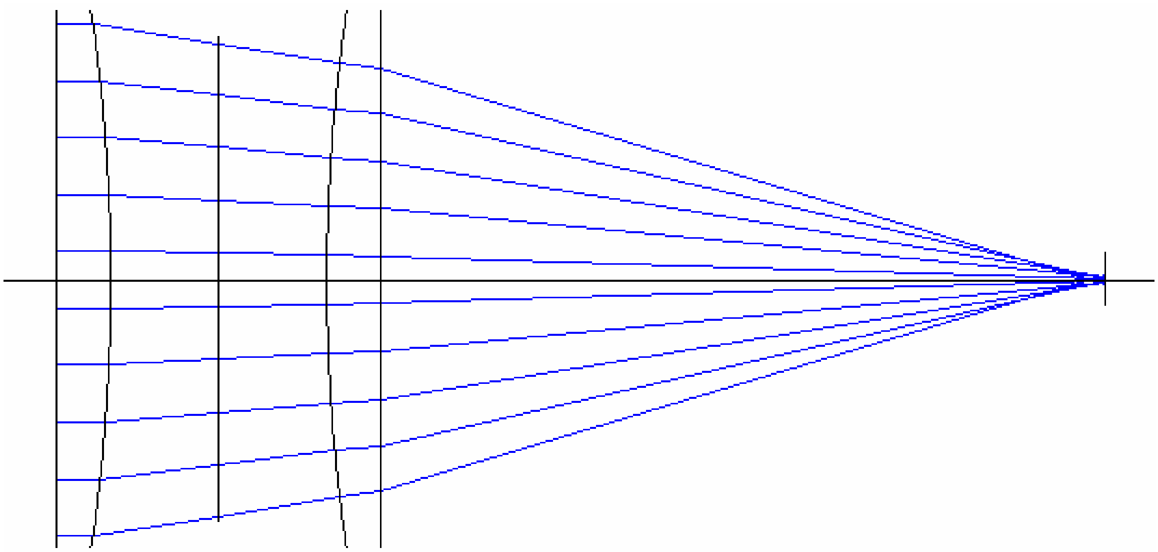

Figure 2-1: A diagram of the starting configuration of the doublet system.

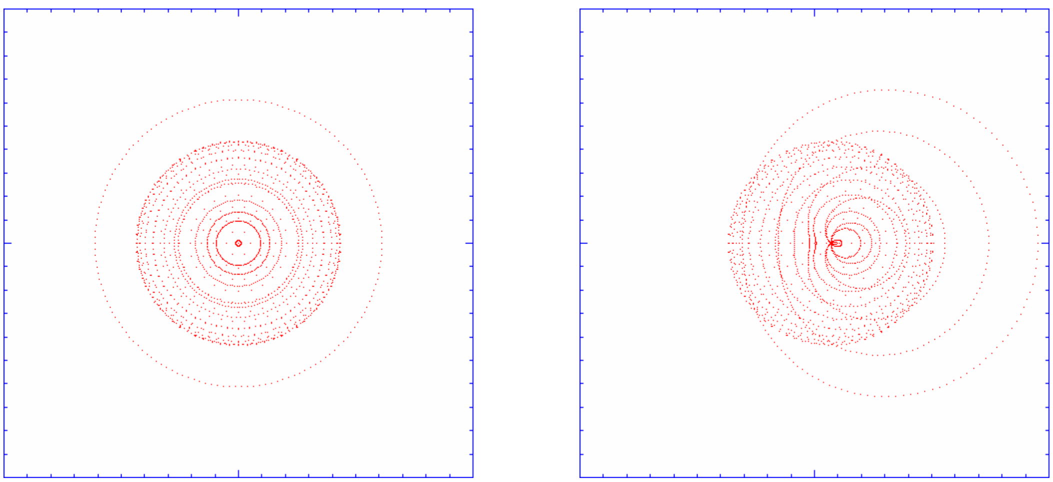

Figure 2-2: Spot diagrams for Case 1 (lenses are plano-convex with spherical surfaces). Left: On-axis, right: one degree off-axis. Full scale is 0.25 cm.

Case 2: Using the variation window, we now search for a better spot size by varying the eccentricity of surfaces 2 and 4. The results are given in Table 2-1 (Case 2). Note that there is a partial degeneracy between the two eccentricities. Spot diagrams are given in Figure 2-3. The spherical aberration is basically gone and residual coma is evident.

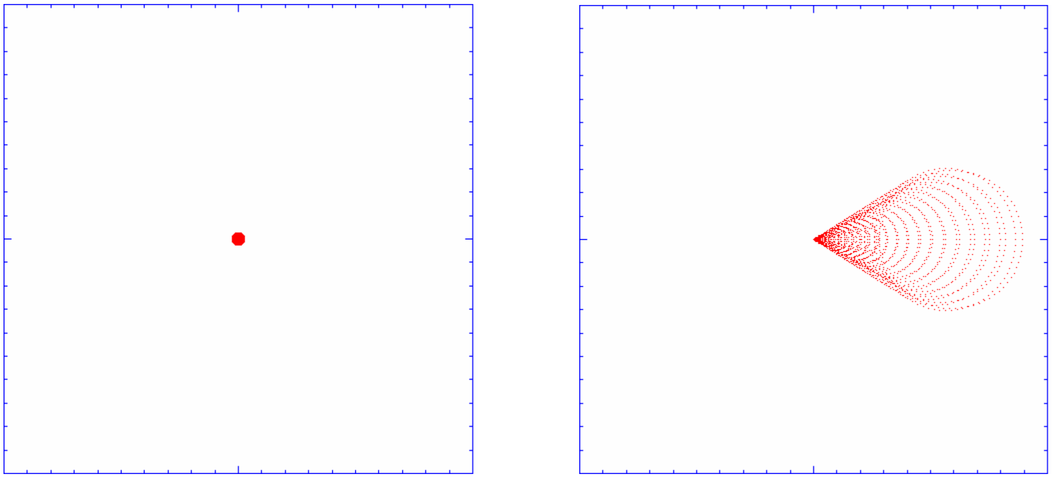

Figure 2-3: Spot diagrams for Case 2 (lenses are plano-convex with conic surfaces). Left: On-axis, right: one degree off-axis. Full scale is 0.1 cm.

Case 3: Now we try optimizing by bending the lenses (starting from Case 1). Initially the search was done with bending to two lenses (surface 1 & 2 together and surface 4 and 5 together) from -0.4 to 0.4 (cm^{-1}) about the nominal (Case 1) values. The search space was then narrowed. The spot diagrams are shown in Figure 2-4 and the results rms spot sizes are list in Table 2-1. A system plot is shown in Figure 2-5.

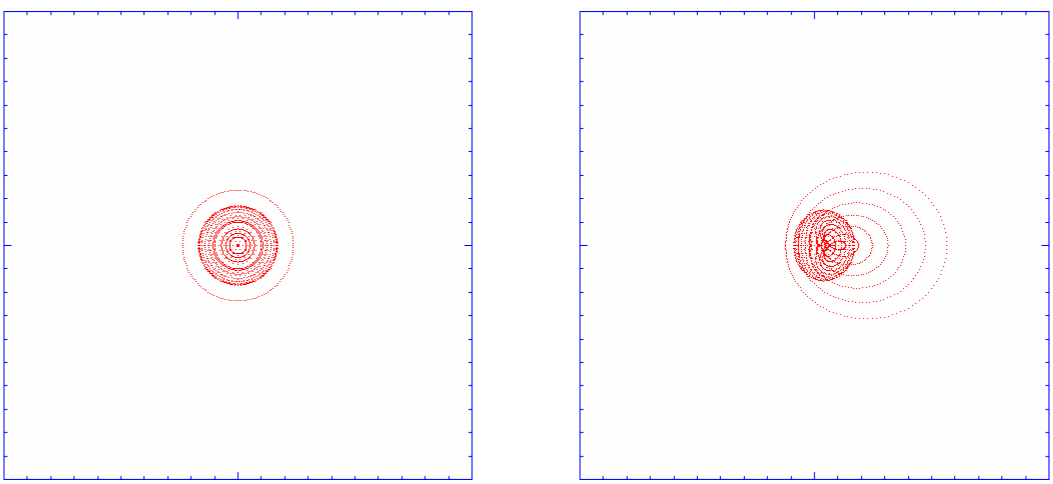

Figure 2-4: Spot diagrams for Case 3 (lenses are bent with spherical surfaces). Left: On-axis, right: one degree off-axis. Full scale is 0.05 cm.

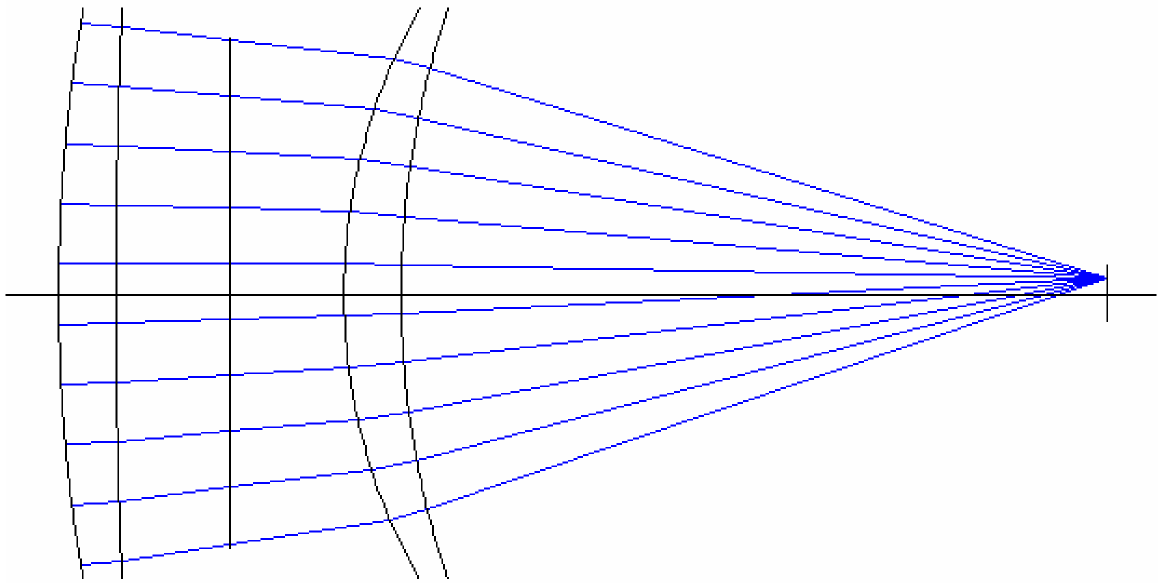

Figure 2-5: Schematic of optical system after bending lenses 1 and 2 to minimize the rms spot size (Case 4).

Case 4: We now vary the eccentricity of the surfaces of the second lens. This basically can be used to get rid of spherical aberration. Figure 2-6 shows the resulting spot diagrams. Not that there is still residual coma in the system. There is also significant field curvature and we have not worried about chromatic aberrations, so this system still has a ways to go. Note that the rms spot size for 1 degree off-axis is about 0.9 arcminutes. Not great but much better than the original doublet.

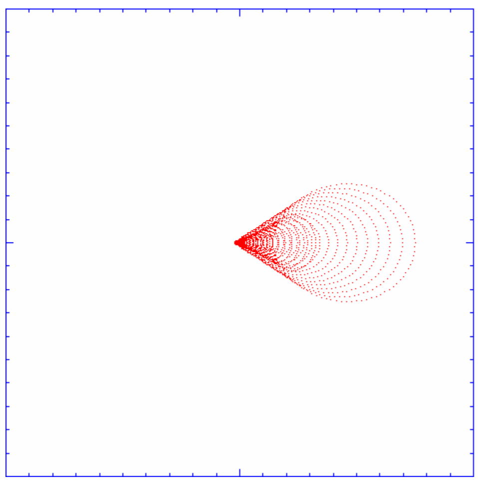

Figure 2-6: Spot diagrams for Case 4 (lenses are bent with conic surfaces). The source is one degree off-axis. The on-axis case gives a near perfect image. Full scale is 0.025 cm.

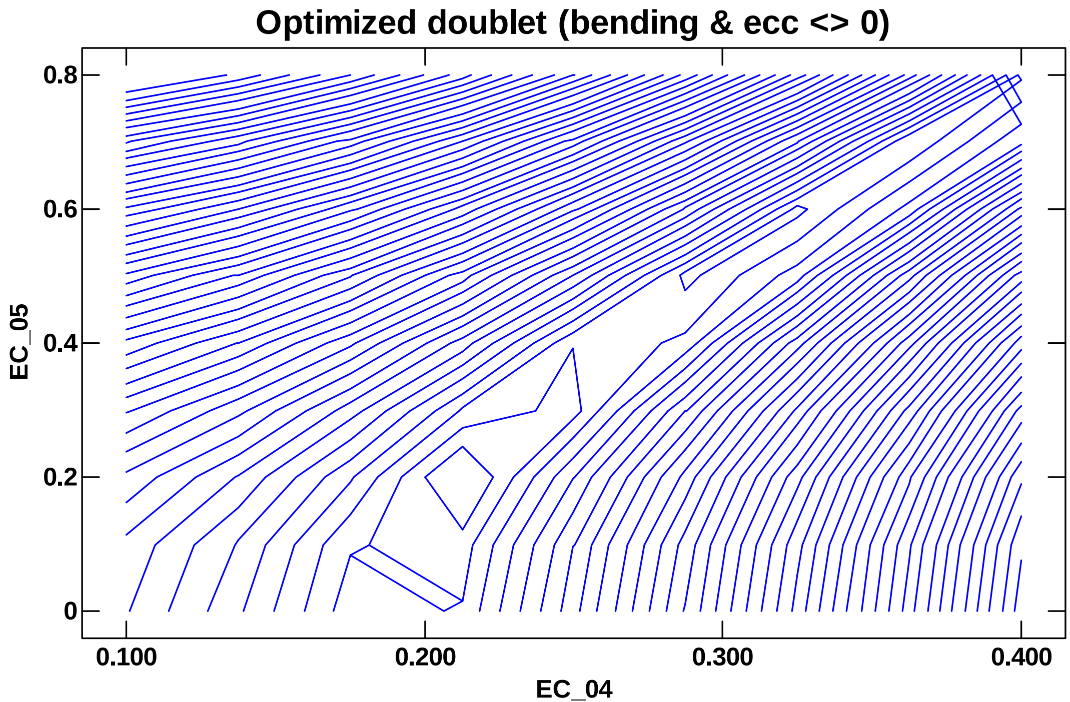

Figure 2-7: Optimization plot of smallest rms vs. eccentricity surfaces 4 and 5 for Case 5. The smallest rms is e = 0.2127 for surface 4 and e = 0.20 for surface 5. This optimization is done for an on-axis source. There is not much difference for a source one degree off-axis.

Ray-trace files used:

File	Description
pc with conic surface (f = 15).ray	Plano-convex lens with eccentricity optimization
cp with conic surface (f = 15).ray	Same for convex-plano lens
doublet (pc-cp, conics) (f = 15).ray	Case 2, set ecc = 0 to get case1
doublet - optimized (f = 15).ray	Case 4, set ecc = 0 to get case 3