

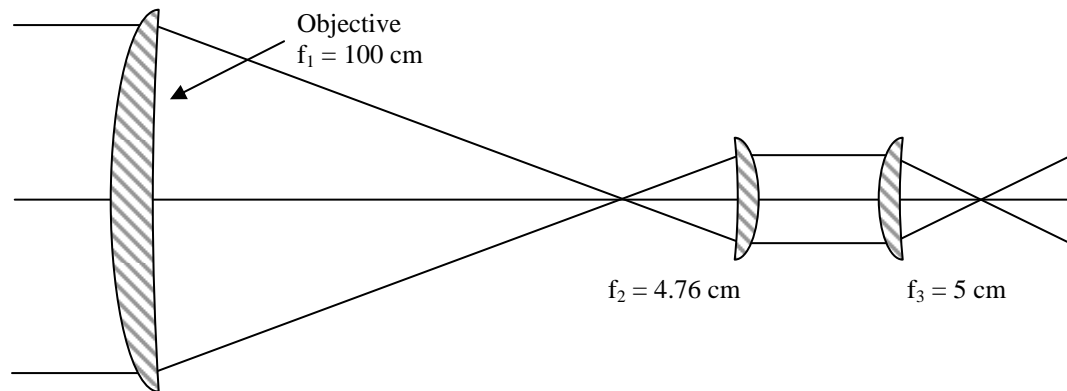
A525 - Problem Set # 4

October 20, 2008

Due: October 27, 2008

The homework assignments will be used as part of your final grade. Please work on this problem set by yourself.

1. A 20-cm diameter, 100 cm focal length lens is followed by two smaller lenses, with focal lengths 4.76 cm and 5 cm, as sketched below. The first of these lies at 104.76 cm past the large lens and the other lies 9.76 cm further on. All of the lenses are made of flint glass ($n = 1.5$). The arrangement is used as a telescope, with the large lens as the objective (primary) and with a detector in the final focal plane. The plot is not to scale.



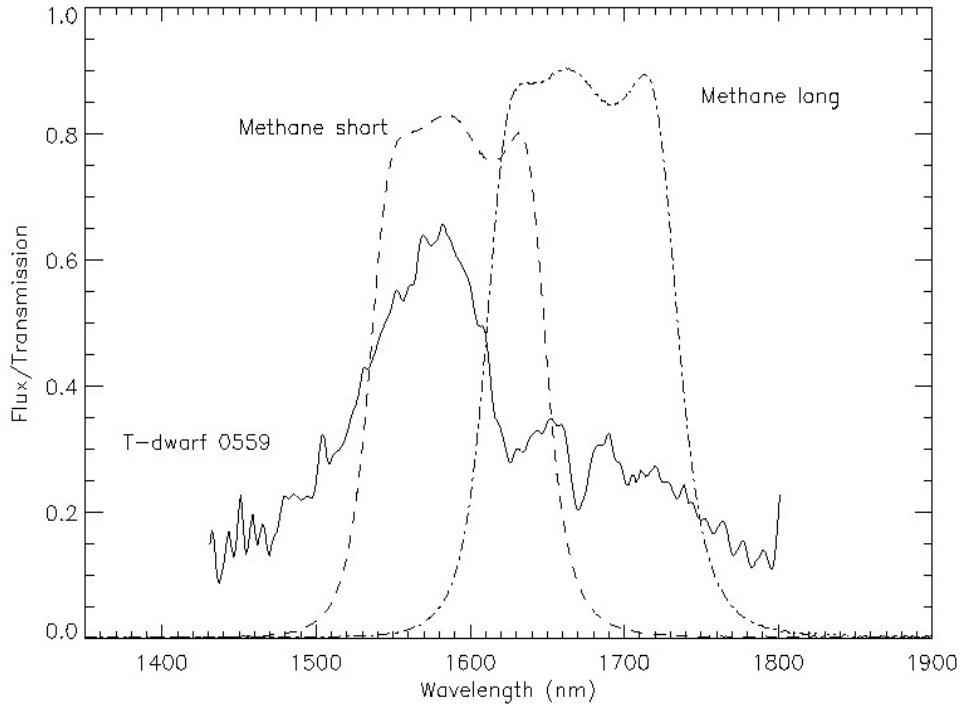
- a) Calculate the plate scale in the focal plane.
- b) Calculate the position and diameter of the pupil that would best serve as a Lyot stop (an image of the objective), and indicate this position in a sketch of the system.

2. Suppose we wish to use a 30-cm telescope operating at $2.2 \mu\text{m}$ (Ks-band) as an acquisition and guiding camera for a larger telescope. The system is diffraction limited (that is the size of a “star” in the focal plane is set by diffraction and not seeing) and we sample $p_{\text{samp}} = 2.5$ pixels across this disk. We only consider point (star-like) sources for guiding. The focal plane has $18.5 \mu\text{m}$ pixels.
- Taking the extraction radius, $\theta_{\text{ext}} = 0.673 \cdot \text{FWHM}$ where FWHM is the Full Width Half Maximum of the diffraction limited point spread function, how many pixels are needed to extract the flux?
 - What is the $f/\#$ of the camera that feeds the focal plane?
 - For typical detector response, and filter and telescope throughputs, the expected source response is 814 e-/s/mJy/m^2 and background intensity at Ks is $2440 \text{ e-/s/arcsec}^2/\text{m}^2$. We will assume that the read noise of the array is zero. Our requirement is a signal-to-noise requirement of 10 in an integration time of 1 second. What is the flux limit (in mJy) of the system?
 - Given a zero magnitude of 667 Jy at Ks, what magnitude is this?
 - Looking at 2MASS data near the South Galactic Pole the source density (# of objects/deg²) for $4 < K_s < 15$ is modeled reasonably well by

$$\rho(< m) = \rho_o e^{\gamma(m-10)} = \rho_o 10^{\gamma(m-10)/2.3}$$

where ρ is the number of sources/deg² with magnitude less than m . For Ks $\rho_o = 17.9$ sources/deg² and $\gamma = 0.743$. Suppose we require that there be 5 stars of sufficient flux density (S/N = 10 in 1 second) in the field of view of the camera. Assuming the array is square, how many pixels across must the array be? Is this a reasonable size?

3. We wish to design an experiment to detect and identify low-mass stars in the Pleiades cluster with the Palomar Wide-field Infrared Camera (WIRC). This will be done by taking exposures in and out of a methane band that is located in the H-band ($1.65 \mu\text{m}$) part of the spectrum. The figure below shows the spectrum of the T5-dwarf 0559 in the H-band, and the two filters we will use, methane short and methane long. The difference between two images (scaled appropriately) in and out of the methane band will show an excess for stars which exhibit the methane band, that is, low-mass stars.



WIRC has an H-band sensitivity of $H = 22.1$, 5-sigma in 1 hour for an M8 V star. The spectral bandwidth of the methane short ($\Delta\lambda_{\text{short}}$) and long ($\Delta\lambda_{\text{long}}$) are respectively 0.4 and 0.48 times the bandwidth of the entire H-band.

The distance of 0559 is estimated to be 10 pc and it has an H-band magnitude of $H = 13.64$. The distance of the Pleiades is 130 pc.

For 0559 we have for the integrals over the bands

$$\frac{CH_4(\text{short})}{H} = 0.54 \quad \frac{CH_4(\text{long})}{H} = 0.41$$

For an M8 V star (which we will take to represent an average star in the cluster with no methane feature), we have

$$\frac{CH_4(\text{short})}{H} = 0.42 \quad \frac{CH_4(\text{long})}{H} = 0.49$$

To look for T-dwarfs, the long wavelength methane band image is scaled so that M8 V stars will subtract to zero in the difference. Stars with an excess of methane short emission relative to methane long are then likely to be T-dwarfs.

What is the integration time required for each band to obtain a signal-to-noise ratio of 5 on the methane decrement (short minus long) on T-dwarfs like 0559 in the Pleiades? (Assume that you expose the methane short and methane long bands for the same amount of time.)