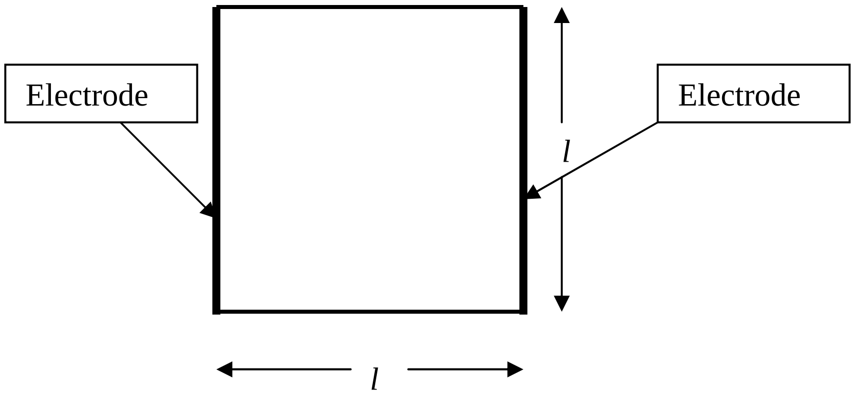
A525 Problem Set # 5

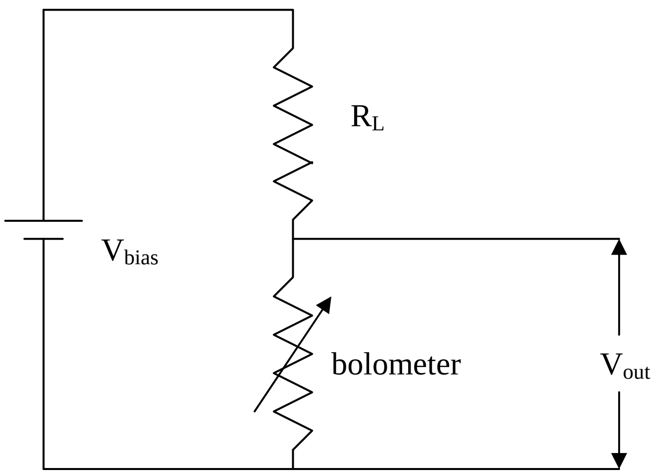
November 14, 2008 Due: November 26, 2008

Problem 1. Imaging that you have a square sheet of conducting metal film of thickness, *t*, and size, *l*. Prove that the resistance of this square metal film as measured from opposite sides (see below) depends only on the thickness of the film, and not on its area. "



Note: you proof should be very short, only a line or two long.

Problem 2. Consider a bolometer operated in the standard biasing and readout circuit illustrated below.



For this exercise, assume $R_L = 10^9\ \Omega$. The bias supply is variable so that we can construct a load curve for the bolometer. The values of V_{out} as a function of V_{bias} are tabulated below.

a). Assuming $\alpha = -2.67$, find G for this bolometer at V_{bias}= 150 Volts.

b). If the frequency response with $V_{out} \sim 0.12$ V extends to 16 Hz, derive the heat capacity of the bolometer.

V_{bias} (V)	**V_{out}(mV)**	**V_{bias} (V)**	**V_{out}(mV)**	**V_{bias} (V)**	**V_{out}(mV)**
10	10.0	**120**	105.1	**230**	158.6
20	19.9	**130**	111.7	**240**	161.8
30	29.7	**140**	117.9	**250**	164.8
40	39.3	**150**	123.7	**260**	167.5
50	48.7	**160**	129.1	**270**	170.1
60	57.8	**170**	134.2	**280**	172.5
70	66.6	**180**	139.0	**290**	174.7
80	75.1	**190**	143.5	**300**	176.8
90	83.2	**200**	147.7	**310**	178.7
100	90.9	**210**	151.6	**320**	180.4
110	98.2	**220**	155.2	**330**	182.1

c). Assuming that the bolometer is operating at 1.5 K and has a detective quantum efficiency, $\eta = 0.9$, what is the Johnson noise limited NEP, NEP_J, for this bolometer at V_{bias} = 150 Volts?

d). What is the thermal noise limited NEP, NEP_T, for this bolometer, again at V_{bias} =150 Volts?

e). What is the zero background NEP for this bolometer at V_{bias} = 150 Volts?

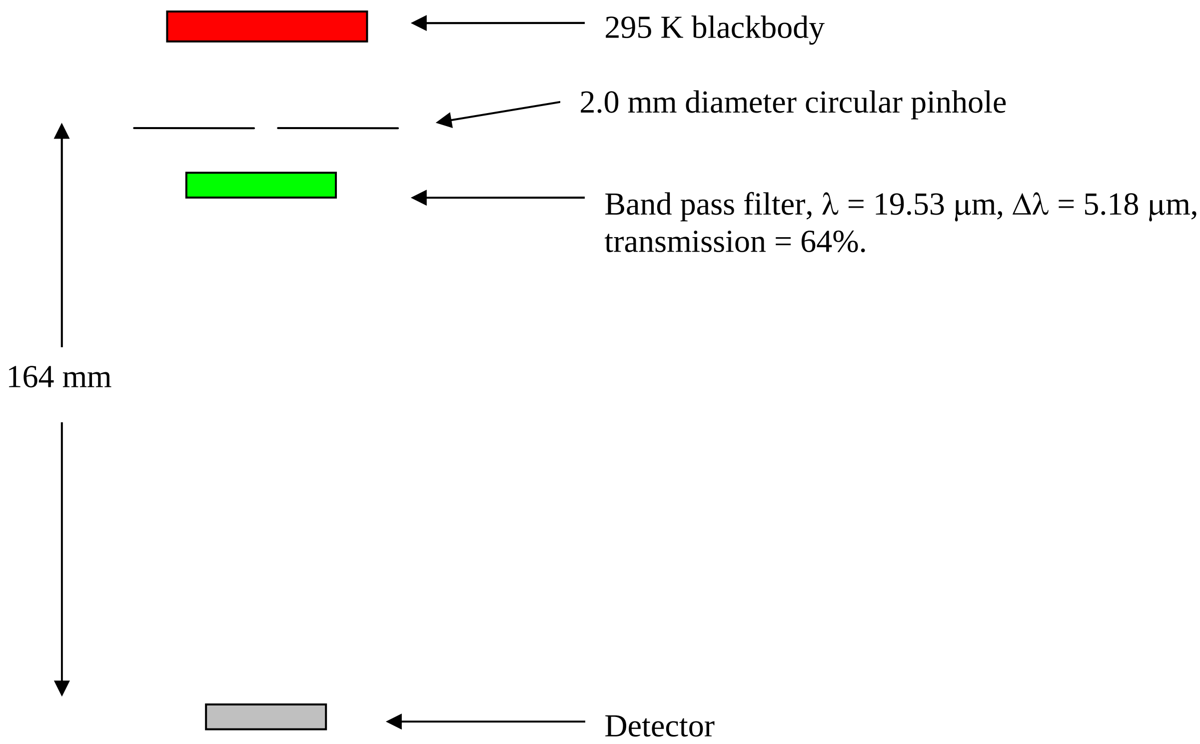
Problem 3. Consider the bolometer from Problem 2, but now imagine that there is excess current noise at the operating frequency in the amount $\langle I^2_{1/f}\rangle^{1/2} = 5 \times 10^{-8} \cdot I_{bolometer} Hz^{-1/2}$.

(a) For $I_{bolometer}$ = 5, 10, 15, 20, 25, and 30×10^{-8} A, derive the zero background NEP for the bolometer. (Hint: see slides 18-19 of bolometer lecture. This noise is analogous to Johnson noise.)

(b) At what bias voltage is the NEP optimal?

(c) Next, assume the bolometer is looking out at a 300 K blackbody with 50% emissivity. The bandwidth that the bolometer accepts is limited by a filter to $\Delta\nu = \nu/R$ where ν is the center frequency of 6×10^{12} Hz. Furthermore, assume that the bolometer is sensitive to two modes of the radiation field, $A\Omega = 2 \cdot \lambda^2$. At what resolving power, R, does the bolometer performance become background limited?

Problem 4. Shown below is a setup for testing infrared detectors. The detector is illuminated through a 2.0 mm pinhole by a 295 K blackbody source. The pinhole is located 164 mm from the detector. A band pass filter centered at 19.53 μm that has a FWHM of 5.18 μm and a transmission of 64% is used to limit the light reaching the detector. The pinhole, filter, and surrounding structure are cold enough so that the only radiation reaching the detector is from the light from the blackbody that passes through the pinhole.



We wish to determine the responsive quantum efficiency (ηG), the gain dispersion (βG), and the detective quantum efficiency (η/β) of the detector. The detector has 50 μm pixels and the gain in the system is 1270 e-/DN, that is 1 "data number" corresponds to 1270 electrons. We will assume that dark current can be ignored.

Two measurements are made, one in the light (pinhole open) and the other in the dark (pinhole closed). The results of the measurements for a single pixel are given below for an integration time of 0.1 second.

Measurement	Signal (DN)	Noise (DN)
Dark	1838	1.997
Light	12963	3.605

a) What is the photon rate per pixel (photons/pixel/second) falling on the detector when the pinhole is open?

b) What is the read noise of the detector (in electrons)?

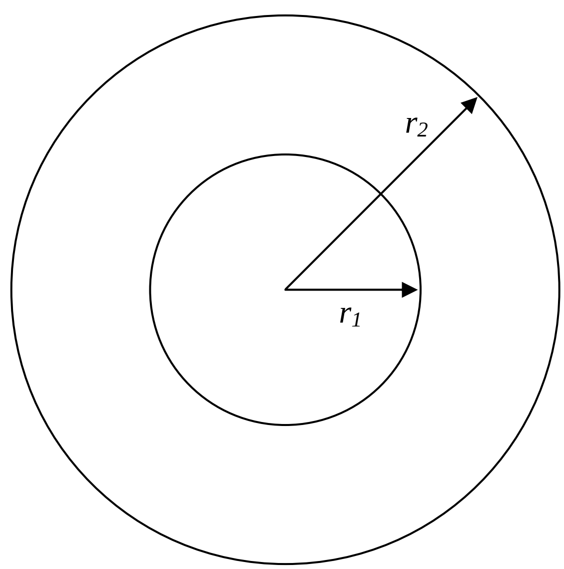
c) Determine the responsive quantum efficiency (ηG), the gain dispersion (βG), and the detective quantum efficiency (η/β) of a pixel.

Problem 5. Consider two concentric spheres as shown below with properties (r_1, ε_1, T_1) and (r_2, ε_2, T_2), which are the radius, emissivity, and temperature respectively, and $r_1 < r_2$.

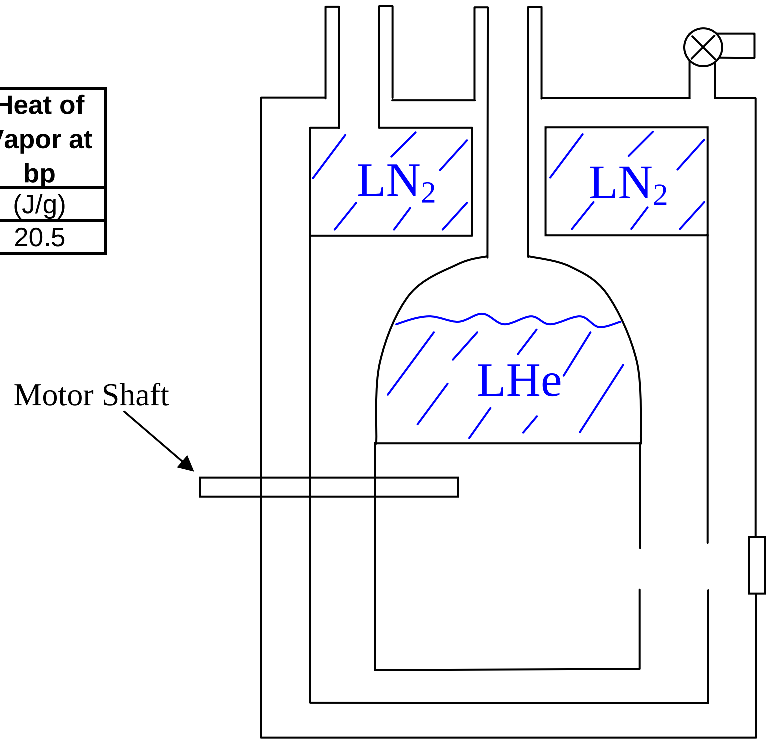
a) Compute the radiative heat transfer between the two spheres.

b) Take the limit as $r_1 \rightarrow r_2$ and show that this approaches the plane parallel radiative heat transfer case we did in class.

[Hint: You probably won't be able to use the multiple bounce technique we used in class for solving the parallel plate case. Set up properly you can use an electrical circuit analogy.]



Cyrogen	Liquid Density at bp	Heat of Vapor at bp
	(kg/l)	(J/g)
He	0.125	20.5



Problem 6: Consider the LN_2-shielded, LHe cryostat shown above. It holds 10 liters of LHe. Assume the cryostat has the following parameters:

- The surface area of the LHe region is 1 m^2.
- The entrance window to the helium can is circular and 5 cm in diameter.
- There are 6 identical motor shafts made of G10 fiberglass (see next page for thermal conductivity properties). Each shaft "touches" the LN_2 shield (so that it is tied at 77 K at that point). The shafts are cylinders 6.4-mm in diameter with a 0.8-mm wall thickness. The length of each shaft from the LN_2 to LHe is 10 cm.
- The emissivities of the outer surface of the LHe "can" and the inner surface of the LN2 "can" are 0.02.
- The temperature of the room (and hence outer can of the cryostat) is 300 K.
- The detector focal plane dissipates 5 mW of power.

  a) Estimate the power entering the LHe volume from the window.

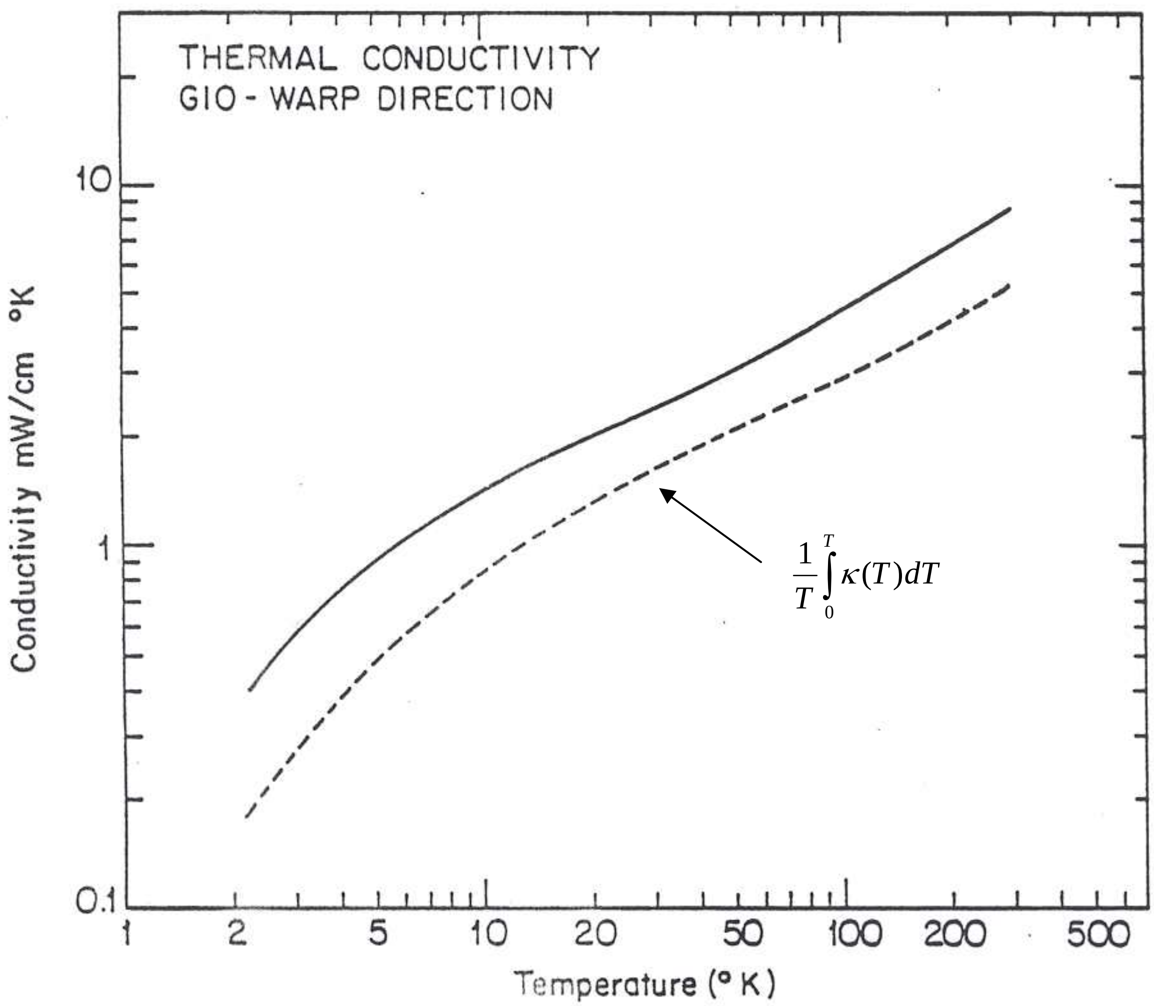
  b) Estimate the power input to the LHe volume via radiative coupling between the LN_2 and LHe cans.

  c) Estimate the heat conducted via one motor shaft to the LHe volume.

  d) Explain why we can ignore conduction of heat into the cryostat by the neck.

  e) What is the total power conducted into the LHe reservoir?

  f) Estimate the hold time for the LHe reservoir.


THERMAL CONDUCTIVITY
G10 - WARP DIRECTION
Conductivity mW/cm °K
10
1
0.1
1
2
5
10
20
50
100
200
500
Temperature (° K)
$\frac{1}{T}\int_0^T \kappa(T)dT$

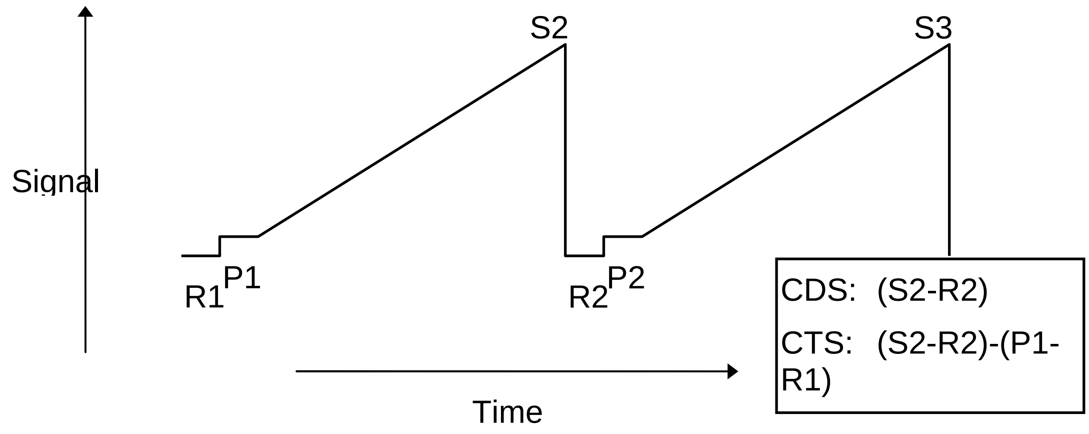


Illustration of signal sampling. R2, P2, and S2 are respectively, the reset, relaxed reset, and signal levels for a given readout of the array. Correlated double sampling (CDS) is given by (S2-R2), correlated triple sampling (CTS) is given by [(S2-R2)-(P1-R1)]. The time between resets is the integration time, while the time between is samples S2 and R2 is called the sample-to-sample or clamp-to-sample time, t_s. For CCDs only the signal and a reference level (like R2) are available. For non-destructive readout infrared arrays, the integration ramp is accessible for a given pixel only when that pixel is accessed (in general once every clock-through of the array). Sample-up-the-ramp (SUR) sampling involves taking samples continuously from points P1 to S2. Multiple sampling, also called Fowler sampling, involves taking many samples near P1 and near S2 (multiple clocking through of the array).

Problem 7: We wish to compute the expected read noise from a CCD array taking data in a correlated double sampling mode (CDS). A schematic illustration of different sampling modes is shown in the diagram above. Since CCDs are only capable of destructive readouts, only CDS sampling can be used. However for infrared arrays, the other sampling modes are applicable.

Let us suppose that the CCD output MOSFET amplifier can be modeled with a white noise and a 1/f noise component, i.e.

$$N_{FET}(f)^2 = W_{FET}{}^2(1 + f_c/f)$$

where W_{FET} is the white noise (V/root-Hz) and f_c is the "corner" frequency of the *1/f* noise component.

To understand how this MOSFET noise translates into a system read noise we must know the "frequency transfer function" for the system. This will depend on the RC filtering and on the "sampling function". Let t_s be defined as the sample time and τ as the RC time constant. The bandwidth, B, is then given by B = 1/(4τ). The response to an RC filter with time constant τ is:

$$g(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{\tau} e^{-t/\tau} & t \ge 0 \end{cases}$$

a) Find the transfer function, $|G(f)|^2$ for the RC filter, where

$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-i2\pi f}\, dt$$

For correlated double sampling, two signals a time t_s apart are differenced. The noise signal is

$$N_{cds}(t) = N(t) - N(t - t_s)$$

where the $N(t)$ and $N(t\text{-}t_s)$ are the filtered CCD noises at time t and $t+t_s$ respectively. The square of the transform of this expression will yield:

$$\left|N_{cds}(f)\right|^2 = \left|N(f)\right|^2 H(f)$$

where *N(f)* is defined like *G(f)*, where H(f) is defined as the sampling function.

b) Find the sampling function, *H(f)*.

The final expression for the noise spectrum is

$$\left|N_{cds}(f)\right|^2 = H(f) \cdot \left|G(f)\right|^2 N_{FET}(f)^2$$

The output noise voltage is given by integrating over all frequencies

$$N_{cds}(V) = \left[\int_0^\infty \left|N_{cds}(f)\right|^2 df\right]^{1/2}$$

This expression can not be generally integrated for arbitrary $N_{FET}(f)$. In what follows below we consider the special case when have only white noise present at the input of the CDS sampler, that is, $f_c = 0$.

c) Derive an expression for $N_{cdsw}(V)$, which is defined as $N_{cds}(V)$ when only white noise is present.

d) What happens when $t_s/\tau >> 1$ and when $t_s/\tau << 1$? Explain why this is the case.

You will notice that the output noise can be made as small as desired by making t_s arbitrarily small. However, this does not maximize the signal-to-noise ratio. Due to the RC filter, the output signal will be given by

$$S_{cds}(V) = S_{ccd}(V)(1 - \exp(-t_s / \tau))$$

where $S_{ccd}(V)$ is the signal voltage of the CCD, and $(1\text{-}\exp(\text{-}t_s/t)) = A_{cds}$ is the signal gain of the CDS processor (Volt/Volt).

e) Derive an expression for the signal-to-noise ratio.

f) How now should t_s be selected to maximize the signal-to-noise ratio?

The read noise of CCD is referred to the "front end" of the device, that is, we must convert 1 e- on the input into a voltage on the output. This conversion is affected by both the capacitance of the CCD node and by the output gain. Let G_o be the gain of the output MOSFET and C_v be the internal voltage response of the CCD (V/e-). The read noise is then given by:

$$RN(e-) = \frac{N_{cdsw}(V)}{C_v A_{cds} G_o}$$

g) Derive the following expression for the read noise (again $f_c = 0$)

$$RN(e-) = \frac{W_{FET}(f)\sqrt{2B}}{C_v G_o (1 - \exp(-t_s / \tau)^{1/2}}$$

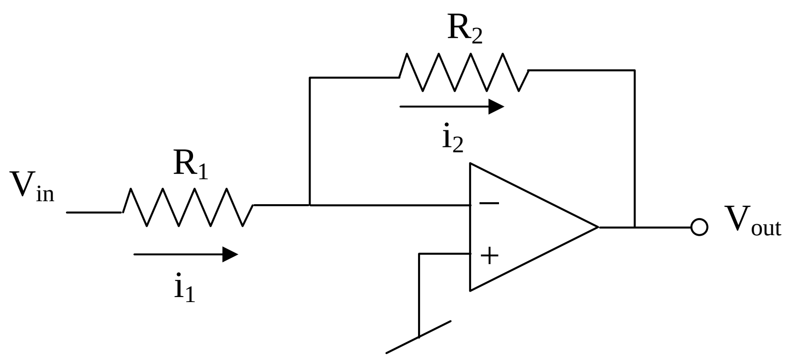
h) Suppose for a CCD camera we have the following parameters: $W_{FET}(f)$ = 6 nV/root-Hz, f_c = 0, C_v = 0.5 μV/e-, G_o = 0.8 V/V, τ = 5 μs. What is the expected read noise for t_s = 5 and 10 μs?

i) Make a plot of read noise vs. t_s ($10^{-7} < t_s < 10^{-3}$ seconds) for $\tau = 10^{-6}$, 10^{-5}, and 10^{-4} seconds using the other parameters defined above.

j) What happens to the read noise at larger values of t_s?

k) Obviously, the larger the value of t_s the longer it takes to read out the CCD. Suppose you had a 2048x2058 CCD, with t_s = 1/50000 seconds. How long would it take to read it out?

Problem 8: Operational amplifiers (op amps) are integrated circuits. For now we don't need to know the details of their insides, but can exam the (idealized) behavior of op amps. Important parameters that we will ignore for now include parameter such as the slew rate (how fast the output can change voltage), noise performance (amplitude vs. frequency), and frequency response. Below is a schematic configuration for a simple amplifier circuit using an op amp.



The idealized op amp behaves in the following manner:

- There are two inputs, - and +. No current can flow into these inputs. They have infinite impedance.
- The op amp will output a current proportional to the voltage difference between the two inputs. If the V- > V+ the op amp will sink current. If V+ > V- the op amp will output a current. Thus in most configuration V- = V+.

Consider the case drawn above. By conservation of charge, we must have $i_1 = i_2$ since no current can flow into the - input. The op amp will "sink" current to keep V- = V+ = 0 V. Thus we have

$$i_1 = \frac{V_{in} - V_-}{R_1} = \frac{V_{in}}{R_1}$$

$$i_2 = \frac{V_- - V_{out}}{R_2} = -\frac{V_{out}}{R_2}$$

So that

$$V_{out} = -V_{in}\frac{R_2}{R_1}$$

The output is amplified by a factor R_2/R_1 and inverted. The configuration above illustrates resistive feedback. Feedback is a very important part of circuit designs using op amps. We won't go into this now, but further information can be found in any introductory electronics text.

a) Consider figure 8a. Perform an analysis similar to that above to determine the relationship between the input and output voltages.

b) Do the same analysis for figure 8b.

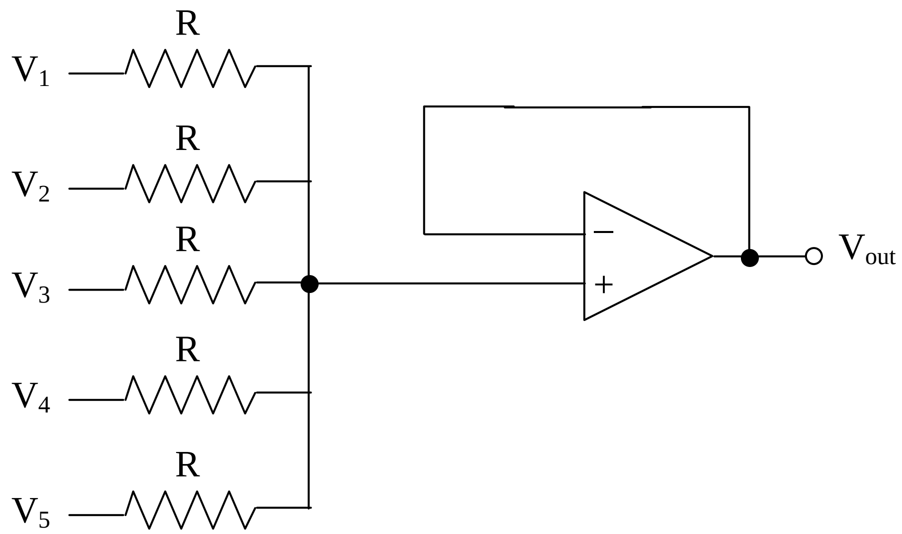


Figure 8a: Determine the output voltage in terms of the input voltages.

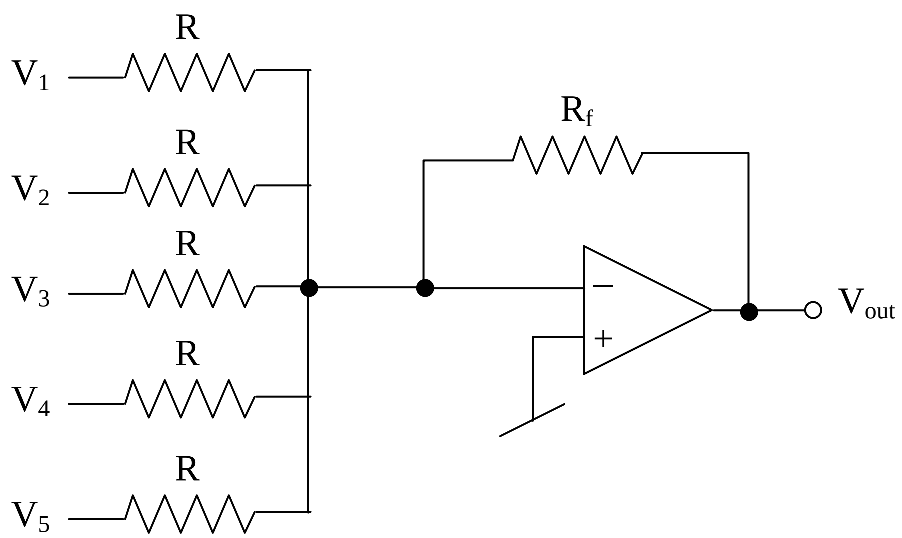


Figure 8b: Determine the output voltage in terms of the input voltages.