Ray Tracing

Astronomy 525

Lecture 02

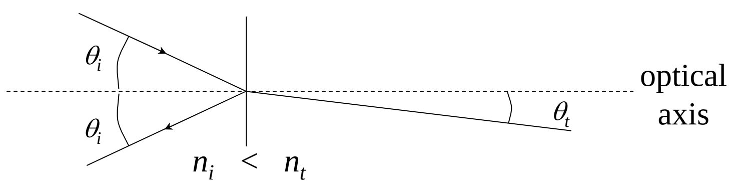
Outline

- Ray Tracing
- Laws of Geometrical Optics
- Sketching Rules
  - Thin Lenses
  - Multiple elements
  - Thick Lenses
  - Mirrors
- Meridional Ray Trace
  - Examples
  - Special Rays
  - Limits
- Aspherics

Ray Tracing

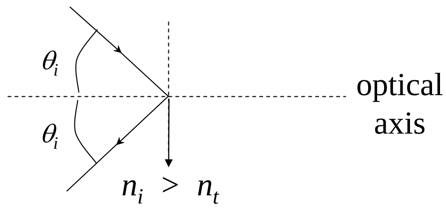
- Ray Tracing -
  - Allows study of the performance of an optical system via geometrical optics.
- Different types of rays:
  - Paraxial - rays very close to the optical axis
  - Marginal - ray at the edge of the entrance pupil
  - Meridional - rays restricted to a plane containing the optical axis
  - Skew - rays traveling in any direction

Laws of Geometrical Optics



- Law of Transmission
  - Light travels in a straight line in a region of constant refractive index
- Law of Reflection
  - The angle of incidence equals the angle of reflection.
- Snell's law
  - $n_i \sin \theta_i = n_t \sin \theta_t$

Total Internal Reflection

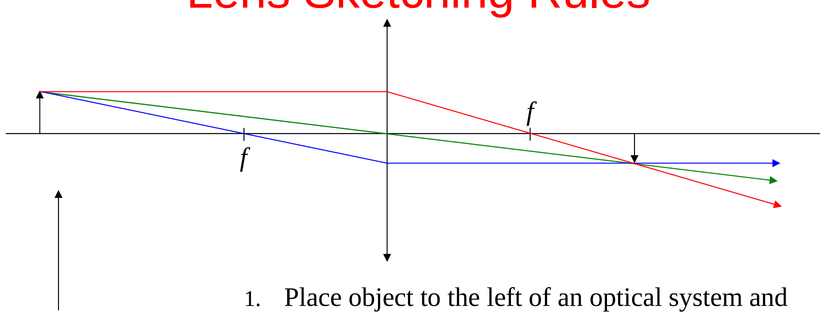


- When $n_i > n_t$ then we can have

$$\sin\theta_t = \frac{n_i}{n_t}\sin\theta_i > 1$$

Which can't happen $\Rightarrow$ **Total Internal Reflection**

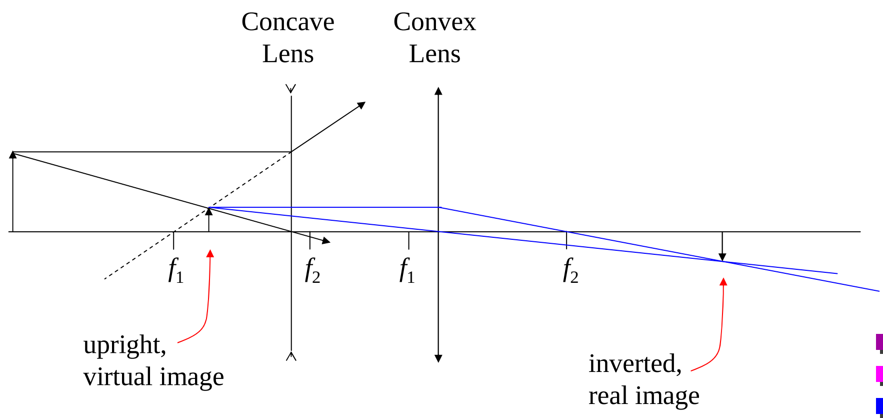
Lens Sketching Rules



Start on left

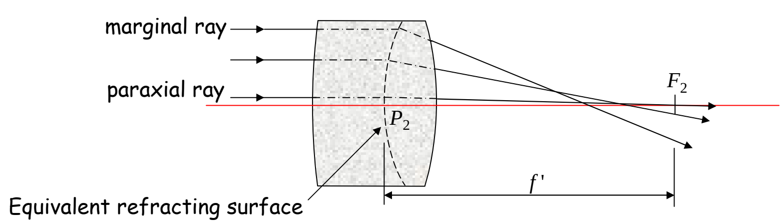
1. Place object to the left of an optical system and trace rays from left to right
2. A light ray parallel to the optical axis will pass through the focus of the lens (red line).
3. A light ray through the focal point will be refracted parallel to the optical axis (blue line).
4. A light ray through the center of the lens is undeviated (green line).

Tracing Multiple Elements



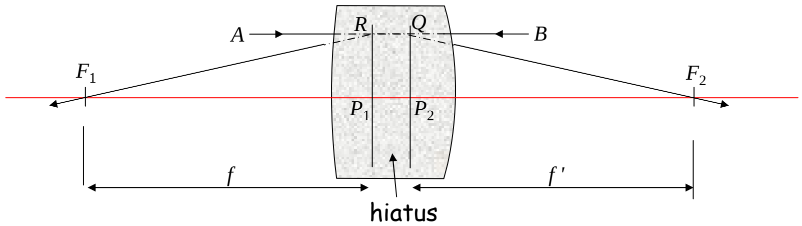
- Combinations or multiple elements:
  - An image of an element becomes the object for the next element.

Thick lenses



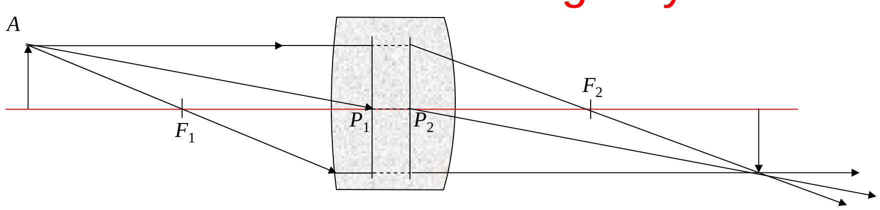
- Consider a set of parallel rays coming into the lens from the left as shown above. They pass through the lens and cross the axis.
- Extrapolating the incoming and outgoing rays until they intersect forms the "**equivalent refracting locus**."
- The plane perpendicular to the paraxial portion of the locus is call the **principal plane** which intersects the axis at the **principal point**, P_2.
- F_2 is the **focal point** and the distance from P_2 to F_2 is the posterior **focal length**, f'.
- For light coming in from the right, there is an equivalent principal plane P_1, focal point F_1, and anterior focal length f.

Thick lens: Properties



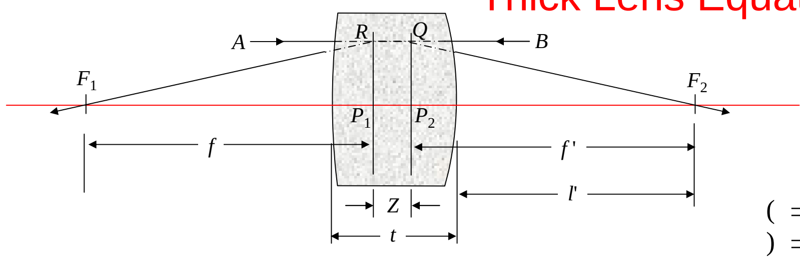
- Consider a paraxial ray, *A* which is effectively bent at the 2nd principal plane. A ray *B* traveling at the same height traveling from the right is effectively bent at *R*.
  - Thus for paraxial rays *R* and *Q* are images of one another.
- A traced ray can be considered to jump across the "hiatus" region between the two principal planes.
- If the indices on either side of the lens are the same then the two focal lengths are the same.
- The **nodal points** are defined such that a paraxial ray directed toward the first nodal point leaves the second with the same slope. For a lens in air, the nodal points correspond to the two principal points.

Thick lens: Sketching Rays



- The same rules apply to thick lenses as to thin, except that there is a transition across the hiatus region.
  - A light ray parallel to the optical axis will pass through the focus of the lens.
  - A light ray through the focal point will be refracted parallel to the optical axis.
  - A light ray through the center of the lens is undeviated – in this case the light leaves P_2 at the same angle as it entered P_1 (if the lens is in air/vacuum).

Thick Lens Equations



$(\Rightarrow r > 0$
$) \Rightarrow r < 0$

focal length: $$\frac{1}{f'} = (n-1)\left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{t}{n}\frac{n-1}{r_1 r_2}\right]$$

n = refractive index
r_1, r_2 = radii of curvature

back focal distance: $$l' = f'\left[1 - \frac{t}{n}\frac{n-1}{r_1}\right]$$

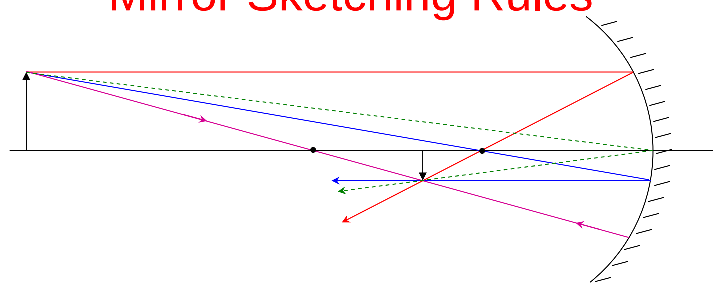
rear principal plane: $$l'_{pp} = l' - f' = -f'\left[\frac{t}{n}\frac{n-1}{r_1}\right]$$

Distance between P_1 & P_2

$Z = t(n-1)/n$ $r_1 \to \infty$ or $r_2 \to \infty$.

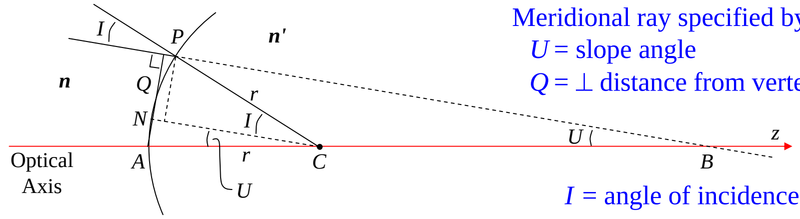
$Z = 0$ when $t = r_1 - r_2$

Mirror Sketching Rules



1. Place object to the left of an optical system and trace rays from left to right
2. A ray parallel to the optical axis will be reflected through the focus of the lens (red line).
3. A ray through the focus will be reflected parallel to the optical axis (blue line).
4. A ray reflected at the vertex makes an equal angle w.r.t. the optical axis (surface normal) as the incident ray (green line).
5. A light ray through the center of the curvature is reflected back on itself (maroon line).

Meridional Ray Trace



Meridional ray specified by:
U = slope angle
Q = ⊥ distance from vertex

I = angle of incidence

I, U, Q = incident ray ← Given these

I', U', Q' = reflected/refracted ray ← Want these (equivalent sketch to above)

CN // BP: $Q = r\sin I + r\sin U$

$$\Rightarrow \boxed{\sin I = \frac{Q}{r} - \sin U} \qquad \text{Eq. (1)}$$

Snell's law: $$\boxed{n\sin I = n'\sin I'} \qquad \text{Eq. (2)}$$

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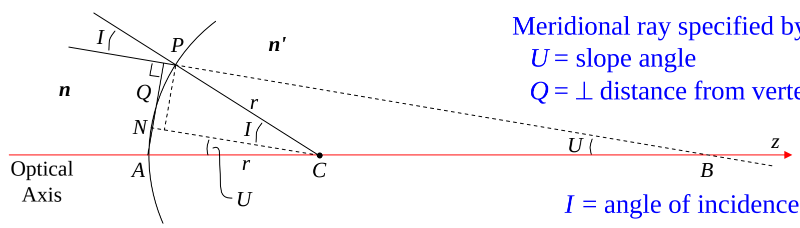
Meridional Ray Trace (cont'd)



Meridional ray specified by:
U = slope angle
Q = ⊥ distance from vertex

I = angle of incidence

∠ PCA: $PCA = I + U = I' + U'$

$$\Rightarrow \boxed{U' = I + U - I'} \qquad \text{Eq. (3)}$$

Primed version of equation 1: $$\boxed{Q' = r\left[\sin I' + \sin U'\right]} \qquad \text{Eq. (4)}$$

Using equations 1-4 we can determine U' & Q' from U & Q of the incident ray and the surface data r, n, and n'.

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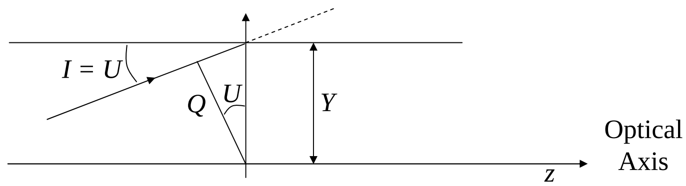
Planar Surface

The case of a planar surface must be considered separately.



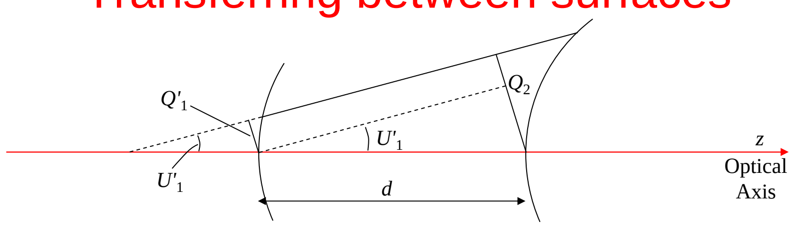
Now $$Y = \frac{Q}{\cos U} = \frac{Q'}{\cos U'} \Rightarrow \boxed{Q' = Q\frac{\cos U'}{\cos U}}$$

This plus Snell's law $$\boxed{\sin U' = \frac{n}{n'}\sin U}$$

Define Q' and U' for a plane.

C = 1/r
= surface curvature
If $C = 0$, use plane surface equations.

Transferring between surfaces



Now : $\boxed{Q_2 = Q_1' + d\sin U_1'}$ and $\boxed{U_2 = U_1'}$

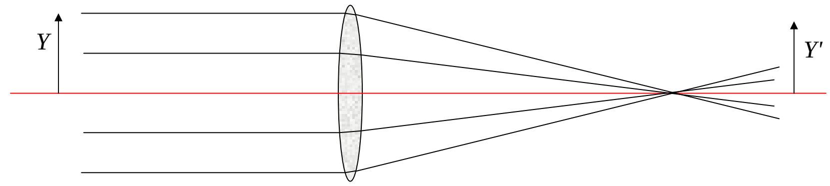
Thus we have Q and U for the next surface, etc.

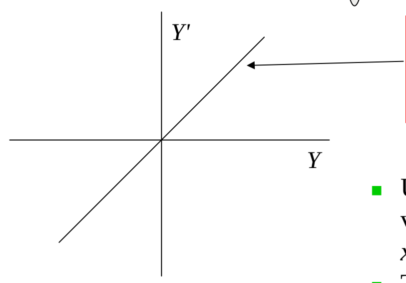
Also need

1) Start-up: transfer from "near-field" or "far-field" object
2) Finish: transfer to focal plane

See Kinglake, pages 24-29.

Examples: Out of Focus

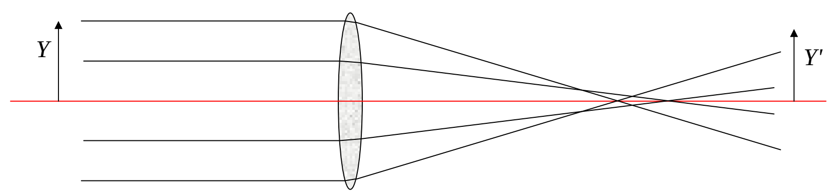


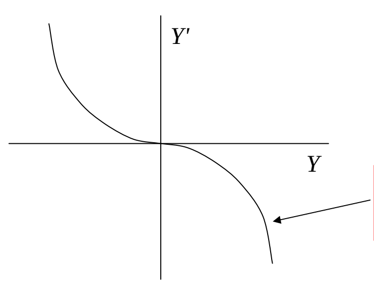


- Slope tells how far out
- Quadrant tells direction (inside or outside)

- Usually the distance, *L*, from the last vertex to the intersection of the ray with *x*-axis is plotted vs. *Y*.
- Thus the focus is automatically “taken out”.

Examples: Spherical Aberration



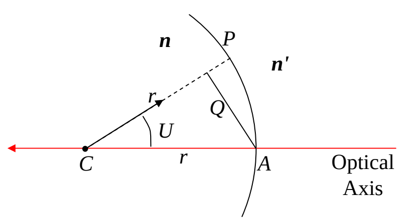


- Get quadratic term for spherical aberration

Special Rays

- Paraxial Ray
  - A ray that always stays near the optical axis
  - small *U*'s and *I*'s $\Rightarrow$ sin $I \to I$, cos $I \to 1$, etc. for *U*
  - Called First Order or Gaussian Optics
  - Linear theory - the ray traces equations now have algebraic solutions.
- Marginal Ray
  - A ray passing through the edge of the first element
  - Spherical aberration can be evaluated with one paraxial and on marginal ray.
  - LA = spherical aberration = L_{marg} - $L_{paraxial}$

Limit: Through radius of curvature



$$(1)\ \sin I = Q/r - \sin U$$
$$(2)\ n \sin I = n' \sin I'$$
$$(3)\ U' = I + U - I'$$
$$(4)\ Q' = r[\sin I' + \sin U']$$

L' = distance of image from vertex

We have:

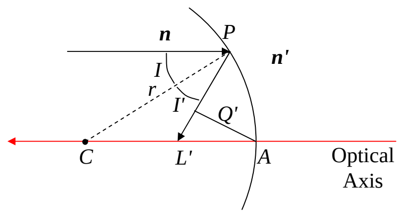
$$\sin U = Q/r$$

$$\Rightarrow \quad I = I' = 0 \quad \& \quad U = U'$$

$$\Rightarrow \quad Q'/r = \sin U'$$

$$\Rightarrow \quad L' = \frac{Q'}{\sin U'} = r \frac{\sin U'}{\sin U'} = r$$

Limit: U = 0, Paraxial ray



(1) $\sin I = Q/r - \sin U$
(2) $n \sin I = n' \sin I'$
(3) $U' = I + U - I'$
(4) $Q' = r\left[\sin I' + \sin U'\right]$

L' = distance of image from vertex

We have (*sin I ~ I*, etc):

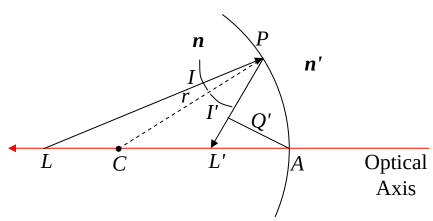
$$I = Q/r \qquad \& \qquad n = -n'$$

$$\Rightarrow \quad I = -I' \quad \& \quad U' = 2I = -2I' = 2Q/r$$

$$\& \quad Q' = r(I' + U') = r(-I + 2I) = rI$$

$$\Rightarrow \quad L' = \frac{Q'}{U'} = r\frac{I' + U'}{U'} = r\left(1 + \frac{I'}{-2I'}\right) = \frac{r}{2}$$

Limit: Paraxial rays



(1) $\sin I = Q/r - \sin U$
(2) $n \sin I = n' \sin I'$
(3) $U' = I + U - I'$
(4) $Q' = r\left[\sin I' + \sin U'\right]$

We have (*sin I ~ I*, etc):

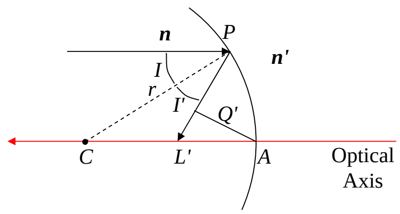
$$I = Q/r - U \quad \& \quad n = -n' \quad \& \quad Q' = r(I' + U') = r(I + U) = Q$$

$$\Rightarrow \quad I = -I' \quad \& \quad U' = 2I + U = 2Q/r - U$$

Now

$$\begin{array}{l} L = Q/\sin U \\ L' = Q'/\sin U' \end{array} \Rightarrow \frac{1}{L} + \frac{1}{L'} = \frac{U}{Q} + \frac{U'}{Q'} = \frac{U}{Q} + \frac{2Q/r - U}{Q} = \frac{2}{r} \equiv \frac{1}{f}$$

Limit: U = 0



(1) $\sin I = Q/r - \sin U$

(2) $n \sin I = n' \sin I'$

(3) $U' = I + U - I'$

(4) $Q' = r\left[\sin I' + \sin U'\right]$

L' = distance of image from vertex

We have:

$$\sin I = Q/r \quad \& \quad n = -n'$$

$$\Rightarrow \quad I = -I' \quad \& \quad U' = 2I = -2I'$$

$$\& \quad Q' = r(\sin I' + \sin U')$$

$$\Rightarrow \quad L' = \frac{Q'}{\sin U'} = r\left(1 - \frac{\sin I}{\sin 2I}\right) = r\left(1 - \frac{1}{2}\frac{1}{\sqrt{1-(Q/r)^2}}\right)$$

Notice that L' depends on Q, the height above the optical axis

Aspheric Surfaces

- An aspheric surface can expressed as a departure from a sphere of curvature, *c (= 1/r)*

$$X = \frac{cY^2}{1 + \left(1 - c^2Y^2\right)^{1/2}} + a_4 Y^4 + a_6 Y^6 + \cdots$$

- If the surface is a conic section then we have

$$X = \frac{cY^2}{1 + \left[1 - c^2Y^2\left(1 - e^2\right)\right]^{1/2}}$$

- Where c is the vertex curvature and e is the eccentricity, $1\text{-}e^2$ is the conic constant

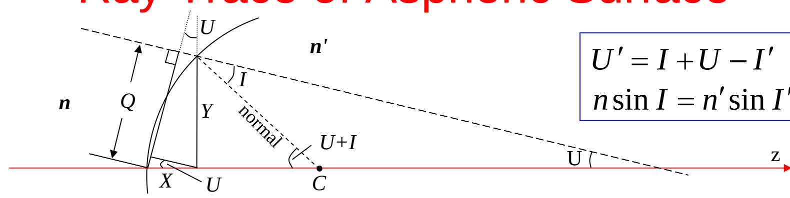
Conic Surfaces

Surface	*Eccentricity*	*Conic Constant*
	e	***1 - e²***
Hyperbola	> 1	< 0
Parabola	1	0
Prolate spheroid (small end of ellipse)	< 1	< 1
Sphere	0	1
Oblate spheroid (side of ellipse)	--	> 1

Ellipse: $e = \frac{\sqrt{a^2 + b^2}}{a}$ Hyperbola: $e = \frac{\sqrt{a^2 - b^2}}{a}$

a = semi-major axis, *b* = semi-minor axis

Ray Trace of Aspheric Surface



$$U' = I + U - I'$$
$$n \sin I = n' \sin I'$$

From the figure above: $Q = X \sin U + Y \cos U$ where $X = X(Y)$

- Now $X = X(Y)$ and the above equation can be solved iteratively to find Y, e.g. using Newton's rule

$$R = X \sin U + Y \cos U - Q \quad \& \quad Y_{new} = Y_{old} - R/(dR/dY)$$

- The slope of the normal is dX/dY, so we have two new equations

$$\tan(U + I) = dX/dY$$
$$Q' = X \sin U' + Y \cos U'$$

for conic sections $\frac{dX}{dY} = \frac{cY}{\left[1 - c^2Y^2(1 - e^2)\right]^{1/2}}$