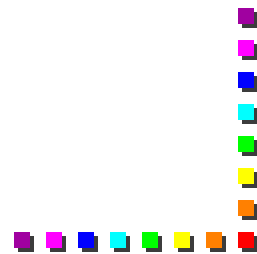


Optical Aberrations

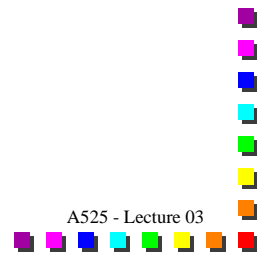
Astronomy 525

Lecture 03

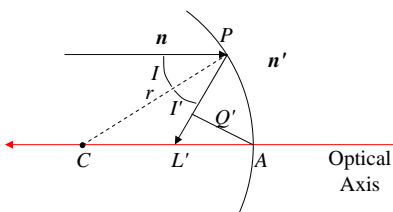


Outline

- Aspherics
- Optical aberrations
 - on-axis
 - off-axis
- Stops
 - aperture stop
 - field stop



Limit: $U = 0$



(1) $\sin I = Q/r - \sin U$

(2) $n \sin I = n' \sin I'$

(3) $U' = I + U - I'$

(4) $Q' = r [\sin I' + \sin U']$

We have:

$$\sin I = Q/r \quad \& \quad n = -n'$$

$$\Rightarrow I = -I' \quad \& \quad U' = 2I = -2I'$$

Notice that L' depends on Q , the height above the optical axis

$$\& \quad Q' = r(\sin I' + \sin U')$$

$$\Rightarrow L' = \frac{Q'}{\sin U'} = r \left(1 - \frac{\sin I}{\sin 2I} \right) = r \left(1 - \frac{1}{2} \frac{1}{\sqrt{1 - (Q/r)^2}} \right)$$

L' = distance of image from vertex

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Aspheric Surfaces

- An aspheric surface can be expressed as a departure from a sphere of curvature, $c (= 1/r)$

$$X = \frac{cY^2}{1 + (1 - c^2Y^2)^{1/2}} + a_4Y^4 + a_6Y^6 + \dots$$

- If the surface is a conic section then we have

$$X = \frac{cY^2}{1 + [1 - c^2Y^2(1 - e^2)]^{1/2}}$$

- Where c is the vertex curvature and e is the eccentricity, $1 - e^2$ is the conic constant

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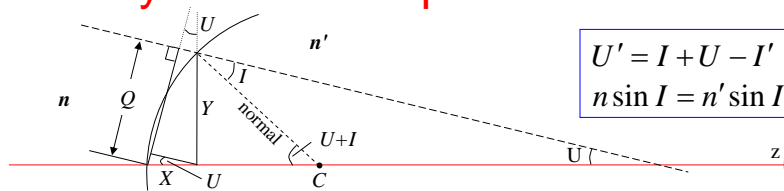
Conic Surfaces

Surface	Eccentricity	Conic Constant
	e	$1 - e^2$
Hyperbola	> 1	< 0
Parabola	1	0
Prolate spheroid (small end of ellipse)	< 1	< 1
Sphere	0	1
Oblate spheroid (side of ellipse)	$--$	> 1

Ellipse: $e = \frac{\sqrt{a^2 + b^2}}{a}$ Hyperbola: $e = \frac{\sqrt{a^2 - b^2}}{a}$

a = semi-major axis, b = semi-minor axis

Ray Trace of Aspheric Surface



$$U' = I + U - I'$$

$$n \sin I = n' \sin I'$$

From the figure above: $Q = X \sin U + Y \cos U$ where $X = X(Y)$

- Now $X = X(Y)$ and the above equation can be solved iteratively to find Y , e.g. using Newton's rule

$$R = X \sin U + Y \cos U - Q \quad \& \quad Y_{new} = Y_{old} - R / (dR / dY)$$

- The slope of the normal is dX/dY , so we have two new equations

$$\tan(U + I) = dX / dY$$

$$Q' = X \sin U' + Y \cos U'$$

for conic sections $\frac{dX}{dY} = \frac{cY}{[1 - c^2 Y^2 (1 - e^2)]^{1/2}}$

Aberrations

- On-axis:
 - Chromatic
 - Spherical
- Off-axis
 - Coma
 - Astigmatism
 - Field curvature
 - Distortion

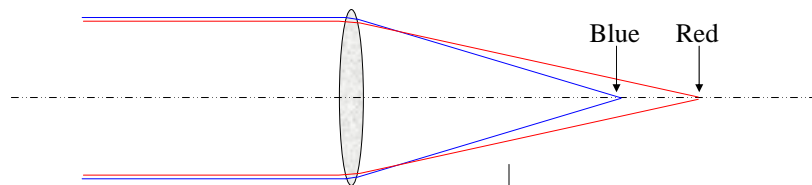
Optical Aberrations

7

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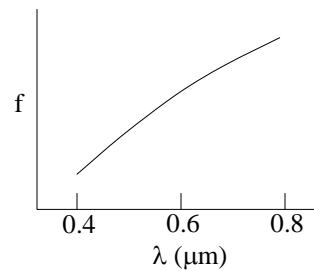
Chromatic Aberration

- Spherical and chromatic aberrations are the only “on-axis” aberrations.
- Chromatic aberration occurs for lenses only.



$$\frac{1}{f} = (n-1) \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$n = n(\lambda)$$



Optical Aberrations

8

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Longitudinal Spherical Aberration

Y

← LA

- Spherical Aberration: Light rays striking the entrance aperture different heights but parallel to the optical axis focus at different places.
- Longitudinal Aberration (LA): $LA = L - L_{\text{paraxial}}$

Optical Aberrations 9 A525 - Lecture 03

Transverse Spherical Aberration

a b c d e

↑ ↑ ↑ ↑ ↑

h

TA

- TA = distance of ray from the axis

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Optical Aberrations

Coma

The effective focal lengths and transverse magnifications differ for rays transversing off-axis regions of the lens

$Coma \propto y^2\theta$

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Optical Aberrations

Coma Diagram

- For an oblique bundle of rays, coma occurs when the intersection of the rays is not symmetrical, that is, shifted w.r.t. the axis of the bundle.
- Focal length and magnification are different for each “ring” on lens.
- Want paraxial and marginal magnifications to be the same.

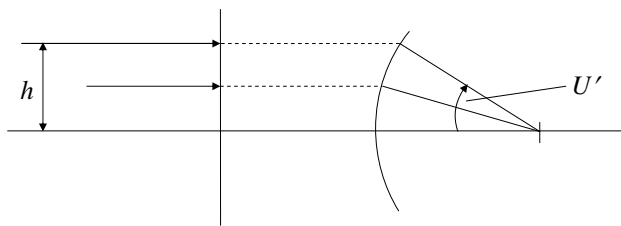
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Abbe Sine Condition

- For an optical system to be free of coma, it must obey the Abbe sine condition (see Basic Optics for derivation) which for a very distant object is:

$$\frac{h}{\sin U'} = C$$

h = height of ray before it enters the system
 U' = angle between ray and optical axis as it travels towards focus
 C = constant



Optical Aberrations

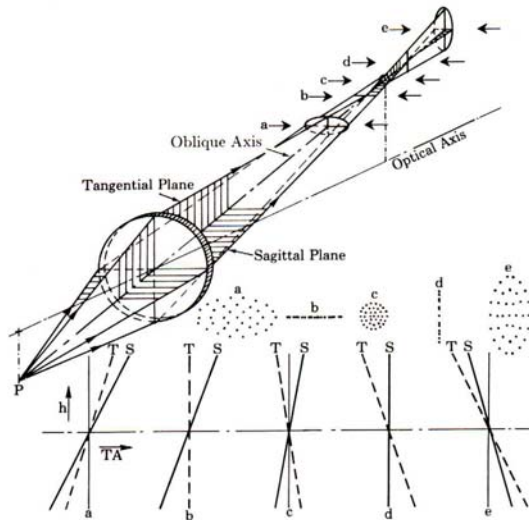
13

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Astigmatism

The foci in the tangential (meridional) and sagittal planes are at different location.



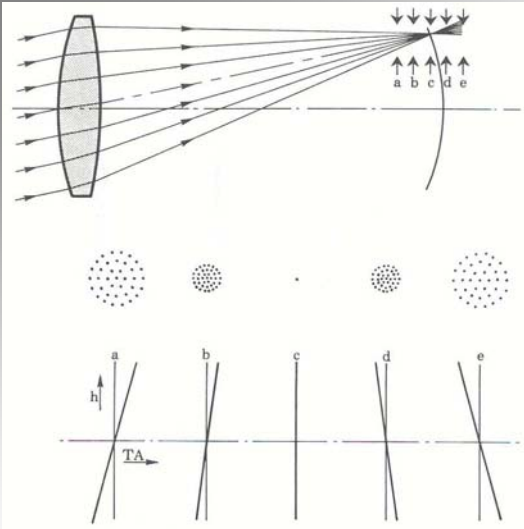
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14

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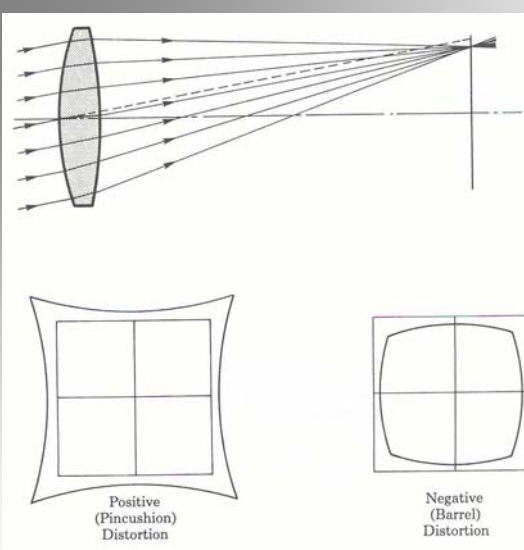
Field Curvature



- With a finite aperture the image plane is a curve.
- Also called Petzval field curvature

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Distortion



- Happens if the transverse magnification is a function of the off-axis image distance.
- Each point may be sharply focused but the image is distorted.
- Introduction of a stop can cause distortion.

Optical Aberrations 16 A525 - Lecture 03

Aberrations

- Expanding beyond the linear approximation give the third-order Seidel aberration terms. The angular aberrations are:

$$AA = a_s \frac{y^3}{R^3} + a_c \frac{y^2 \theta}{R^2} + a_a \frac{y \theta^2}{R} + a_{fc} \frac{y \theta^2}{R} + a_d \theta^3$$

↑ spherical aberration
 ↑ coma
 ↑ astigmatism
 ↑ field curvature
 ↑ distortion

AA = angular aberration (e.g. arcsec or radians)

a_i = constants

R = radius of curvature

y = height of ray

θ = angle of incidence of rays from object at infinity



Aberrations (cont'd)

- Taking $R = f_n y$ then:

$$AA_{max} = \frac{a_s}{f_n^3} + a_c \frac{\theta}{f_n^2} + a_a \frac{\theta^2}{f_n} + a_{fc} \frac{\theta^2}{f_n} + a_d \theta^3$$

↑ spherical aberration
 ↑ coma
 ↑ astigmatism
 ↑ field curvature
 ↑ distortion

$AA_{max} = AA$ for $y = y_{max} = D$

f_n = f-number

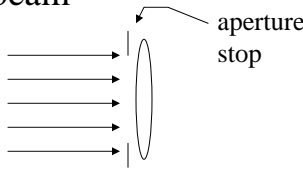
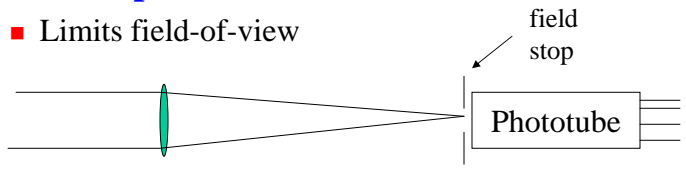
D = aperture diameter

Note: the "faster" the optical system, the greater the aberrations!



Stops

- A stop is something in the optical system that limits the diameter of the beam
- **Aperture stop**
 - Limits amount of light
 - Like a camera iris
- **Field stop**
 - Limits field-of-view

Optical Aberrations
19
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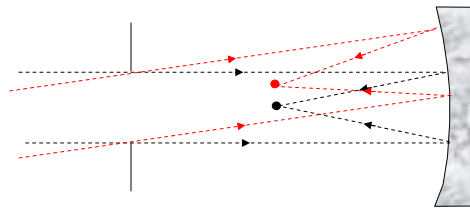
Stops and Throughput

- The throughput, $A\Omega$, of the optical system is set by the two stops:
 - A - by aperture stop
 - Ω - by field stop
- The position of stops can affect system aberrations.

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20
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Stop at center of curvature

- Consider a spherical mirror with an aperture stop at the center of curvature



- The “on-axis” and “off-axis” beams pass around the center of curvature and hit the mirror. There is no “optical axis” for a sphere so there are no “off-axis” rays.
- No off-axis aberrations -- just spherical aberration!

Optical Aberrations

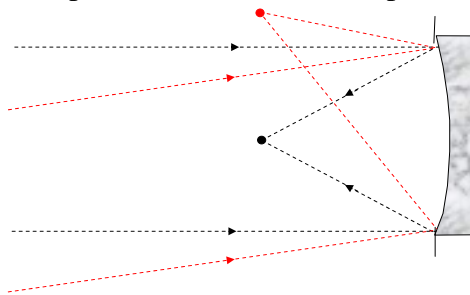
21

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Stop at mirror

- Consider a spherical mirror with an aperture stop at mirror



- There is now an axis defined by the line from the center of the stop (center of the mirror) to the center of curvature.
 - Off-axis \Rightarrow coma
- The location of the aperture stop controls aberrations.

Optical Aberrations

22

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Stops and Distortion

The position of a stop can affect distortion.

Optical Aberrations 23 A525 - Lecture 03

Stops and Distortion (cont'd)

Placing the stop symmetrically eliminates distortion (and coma).

Optical Aberrations 24 A525 - Lecture 03

Stops and Vignetting

Common focal plane

Objective L_1 Eyepiece L_2

f_1 f_2

Exit pupil E

Eye E_0

- If your eye is placed next to the eyepiece (E_0), you don't see the whole field. This FOV is vignetted.
- Put your eye at E (the exit pupil) to see the whole field.
 - But eyepiece must be large!

Optical Aberrations
25
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Stop and Field Lens

f_1 f_2

Objective L_1 Field lens L_3 Eyepiece L_2

Exit pupil E

- **Field lens**
 - Place a lens at L_3 (common focus) which reimages L_1 onto L_2 .
 - The field lens does not change the intermediate image
 - In practice, don't put exactly at focus (dust, etc.)
- Now your eye can be next to the eyepiece.

Optical Aberrations
26
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Some References

- Telescope Optics: Evaluation and Design
 - Harrie Rutten and Martin van Venrooij
- Astronomical Optics
 - Daniel Schroeder
- Reflecting Telescope Optics
 - R. N. Wilson
- Optics
 - Hecht and Zajac
- Principles of Optics
 - Born and Wolf

