Grating Spectrometers: Echelle Gratings

Astronomy 525

Lecture 09

Outline

Echelle Spectrometers

- Free spectral range
- Echelle Efficiencies
- Design Parameters
- Example: GEDI – near IR planet finder
- More examples (not covered in class)
  - KEGS – a mid-IR echelle grating spectrometer
  - ZEUS – a submillimeter echelle grating spectrometer

Echelle Gratings

- An echelle grating is a grating that is coarsely ruled, so that the distance between rulings is greater than the wavelength of interest. It is used at large angles of incidence, and refraction.
- Recall the expression for the (slit limited) resolving power of a grating:

$$R_s = 2 \cdot \tan\alpha \cdot G_{proj} / \{\theta_{Beam} \cdot D_{Tel}\}$$

- For an echelle in Littrow mode, α, $\beta = 63.5° \Rightarrow \tan\alpha = 2$ (This is called an **R2 echelle**, since $\tan\alpha = 2$, For a first order grating, typically $\tan\alpha = ½$)
- Therefore, for a given collimated beam size, an echelle spectrometer delivers a higher resolving power. It can also cover a broad range of wavelengths.

Free Spectral Range: I

- If a grating is operated in echelle mode, resonating at wavelength λ in order n, then it will also resonate at the same angle at another wavelength, λ' in an order $n+1$ where the relationship between orders and wavelengths is given by:

$$n\lambda = d \cdot (\sin\alpha + \sin\beta) = (n + 1)\lambda'$$

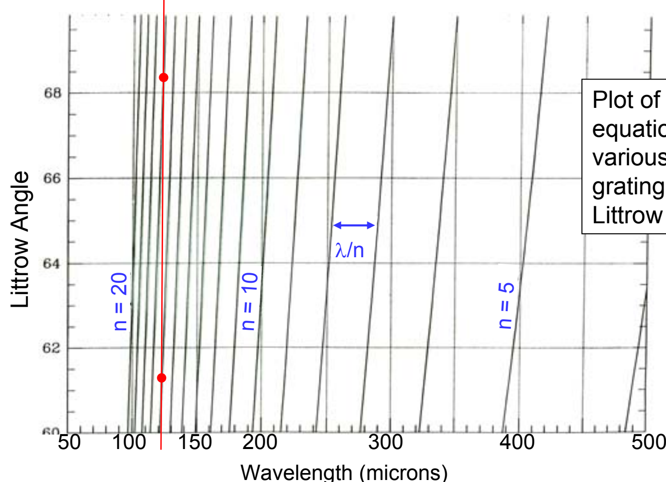
α (β) = incidence (diffracted) angle w.r.t grating normal

- The difference between these two resonant wavelengths is:

$$\Delta\lambda = \lambda - \lambda' = \lambda'/n$$

- $\Delta\lambda$ is called the free spectral range, and is the wavelength interval covered by one order in the echelle without aliasing
- Notice the simple relationship $n\lambda = (n + 1)\lambda'$, which is quite useful for quick analysis.

Free Spectral Range: II



Plot of the grating equation solutions for various orders for a grating operating in Littrow mode.

- Notice that more than one order can be available for a given λ, especially at the shorter λ (largest n)

Echelle Efficiency: I

- It is very important to find the efficiency of diffraction over the various wavelengths in a given free spectral range.
- Recall:

$$I = I_o \{\sin(N\gamma)/N\sin(\gamma)\}^2 \cdot \{\sin(\gamma')/(\gamma')\}^2$$

$$\propto IF \cdot BF$$

BF ← Blaze Function

IF ← Interference Function

where:

$\gamma = \pi d/\lambda \cdot (\sin\beta + \sin\alpha)$ d = space between slits

$\gamma' = \pi b/\lambda \cdot (\sin\beta + \sin\alpha)$ b = width of slit

- The IF is a maximum at $\gamma = m\pi$ (as before), and $IF_{max} = 1$
- The BF is a single slit diffraction pattern, with a maximum at $\gamma' = 0$ (n = 0 in the grating equation)

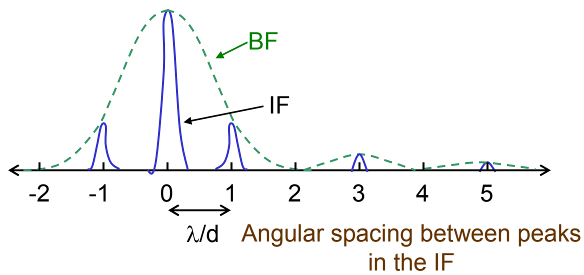
Echelle Efficiency: II

- We saw before that in order to increase the efficiency in a dispersed order ($n \neq 0$), it is necessary to tilt the facets by an angle ϕ (*blazing,* recall that ϕ is the angle of the facet normal w.r.t. the grating normal), so that:

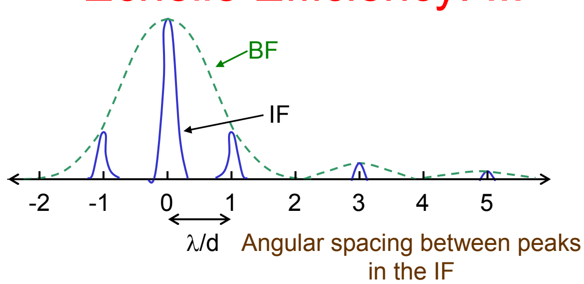
$$\gamma ' = \pi b \cos \phi / \lambda \cdot [\sin(\beta - \phi) + \sin(\alpha - \phi)]$$

- The maximum in the BF occurs for:

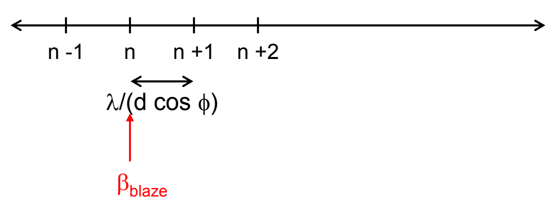
$$\gamma ' = 0 \Rightarrow \alpha + \beta = 2\phi \qquad \text{(Blaze } \lambda)$$



Echelle Efficiency: III



- Blazing shifts the BF by n orders:



Which order to use?

- We wish to evaluate the BF at the n + 1, and n – 1 orders. One can show:

  for $\Delta\beta = \lambda/(d \cos\phi)$: BF = 0.405 (from the BF) (1)
- From the Grating Equation: $n\lambda = d\cdot(\sin\alpha + \sin\beta)$

  $\Rightarrow \quad n\,\Delta\lambda/d = \cos\beta\,\Delta\beta$ (2)
- Combining equations (1) and (2) we have:

  $n\,\Delta\lambda/d = \lambda/(d \cos\phi)\cdot\cos\beta$
- Littrow mode ($\phi = \alpha = \beta$) $\Rightarrow \Delta\lambda = \lambda/n$, which we identify as the free spectral range. Therefore, the efficiency at the edges of the free spectral range are about 40% in the blaze function.

Conclusion: We ought to change orders when we observe a wavelength near the edge of the FSR.

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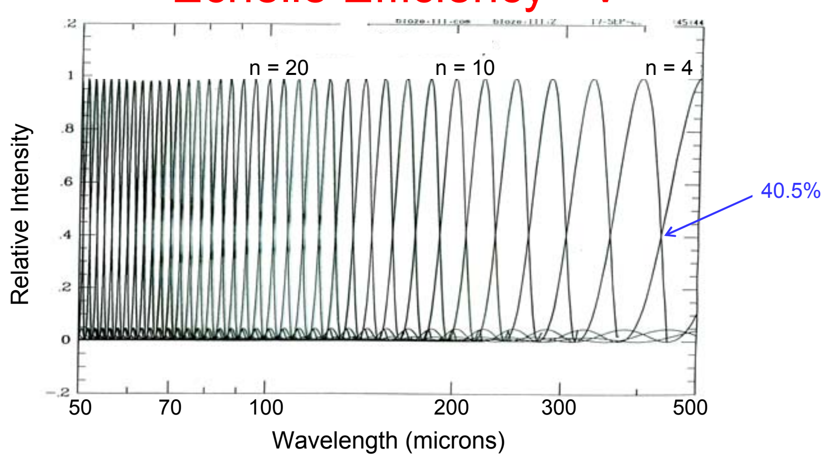
Echelle Efficiency - V



- Relative intensity (efficiency) as a function of λ for various orders, n
- Notice how the orders cross at 40.5% efficiency

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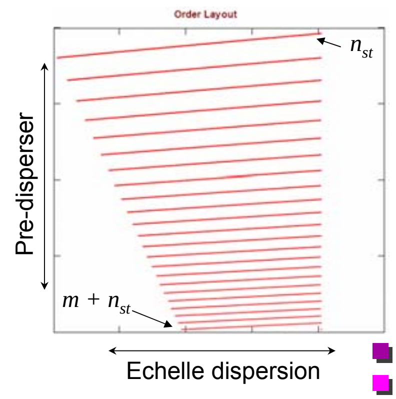
Cross Dispersion

- Different orders of the echelle can be separated with a predisperser, such as a grating, prism, or grism.
- The number of orders required is determined by:

$$n_{st}\lambda_2 = (n_{st}+m)\lambda_1$$

$$\Rightarrow m = \frac{\lambda_2-\lambda_1}{\lambda_1} n_{st}$$

n_{st} = starting order
m = # of orders



- The starting order can be shown to be:

$$n_{st} = \frac{p_{samp}}{N_{pe}} R$$

p_{samp} = samples/$\Delta\lambda_R$
N_{pe} = # pix in echelle direction

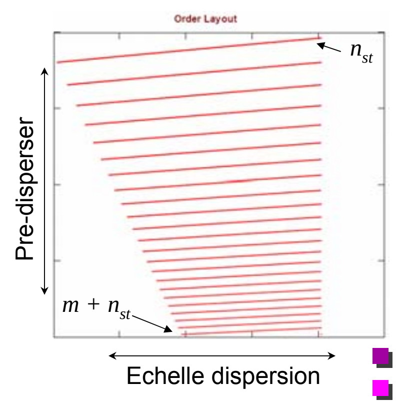
$$\Delta\lambda_{FSR} = \frac{\lambda}{n} \; ; \; \Delta\lambda_R = \frac{\lambda}{R} \; ; \; \frac{N_{pe}}{p_{samp}} = \frac{\Delta\lambda_{FSR}}{\Delta\lambda_R} \Rightarrow n = \frac{p_{samp}}{N_{pe}} R$$

Gap Between Orders

- Of importance is the smallest gap between orders which occurs at the shortest wavelength
- For a 1st order grating pre-disperser, at the shortest wavelength :

$$\lambda_1 = d_g(\sin\alpha_g + \sin\beta_1)$$

$$\Rightarrow \Delta\lambda = d_g \Delta\beta \cos\beta_1$$



- Where $\Delta\lambda = \lambda/(n_{st}+m)$

$$n_{gap} = \frac{f_{cam}\Delta\beta}{s_{pix}}$$

s_{pix} = pixel size

$$\Rightarrow n_{gap} \sim \frac{\lambda_1}{n_{st}+m} \frac{1}{d_g} \frac{f_{cam}}{s_{pix}} \frac{1}{\cos\beta_1} \quad \text{(pixels)}$$

- Slit must be smaller or order overlap occurs

Echelle Groove Spacing

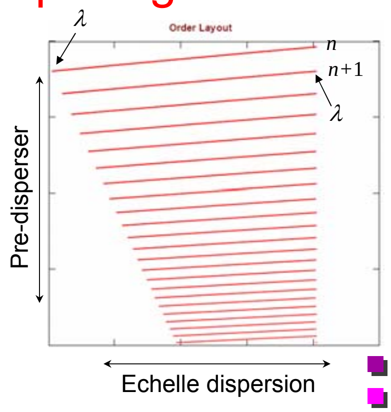
- Using the grating equation for two successive orders at the same wavelength, then differencing gives

$$\frac{\lambda}{d_e} = \sin\beta_{n+1} - \sin\beta_n$$

$$= 2\sin\frac{\beta_{n+1}-\beta_n}{2}\cos\frac{\beta_{n+1}+\beta_n}{2}$$

- Let ϕ be echelle blaze angle and define: $\beta_1 = \theta_1 + \phi$ and $\beta_2 = \theta_2 + \phi$.
- Taking the lowest order, θ_e = max angle of echelle, and $\theta_1 \sim -\theta_2$ then

$$\Rightarrow\ d_e = \frac{\lambda_2}{2\cos\phi\sin\theta_e}$$



- Note that $\theta_e \cong \frac{s_{pix}N_{pe}}{2f^{\#}_{cam}G_{proj}} = \frac{N_{pe}}{p_{samp}R}\tan\phi$

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Predisperser Groove Spacing

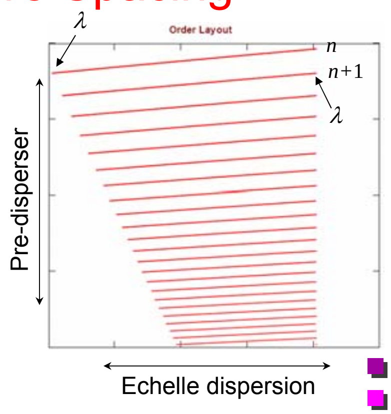
- For 1st order grating predisperer use the grating equation at the ends of the λ range (λ_1, λ_2), then difference

$$\frac{\lambda_2-\lambda_1}{d_g} = \sin\beta_{2g} - \sin\beta_{1g}$$

$$= 2\sin\frac{\beta_{2g}-\beta_{1g}}{2}\cos\frac{\beta_{2g}+\beta_{1g}}{2}$$

- Let ϕ_g be grating blaze angle and define: $\beta_{1g} = \theta_{1g} + \phi_g$ and $\beta_{2g} = \theta_{2g} + \phi_g$.
- Taking the lowest order, θ_g = max angle of grating, and $\theta_{1g} \sim -\theta_{2g}$ then

$$\Rightarrow\ d_g = \frac{\lambda_2-\lambda_1}{2\cos\phi_g\sin\theta_g}$$



- Note that $\theta_g \cong \frac{s_{pix}N_{pg}}{2f^{\#}_{cam}G_{proj}} = \frac{N_{pg}}{p_{samp}R}\tan\phi$

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Example: GEDI Spectrograph

Gemini Exoplanet Discovery Instrument

- Key Specs
  - $R = \lambda/\Delta\lambda = 25{,}000$
  - Cover 0.9 – 1.8 μm, simultaneously
- Sampling
  - ~ 3 pixels per spectral resolution element
- Number of input beams
  - 4 = (2 beams of EDI)×(1 source + ref. beam)
- Choices
  - One 2048x2048 focal plane array
    - Cost factor – but there is enough spectral coverage w/ one
  - Cool spectrograph to ~ 180 – 200K
    - To keep spectrograph background << stellar photon rate
  - Fiber feed with surrounding guide fibers
    - Two bundles on sky for source and background

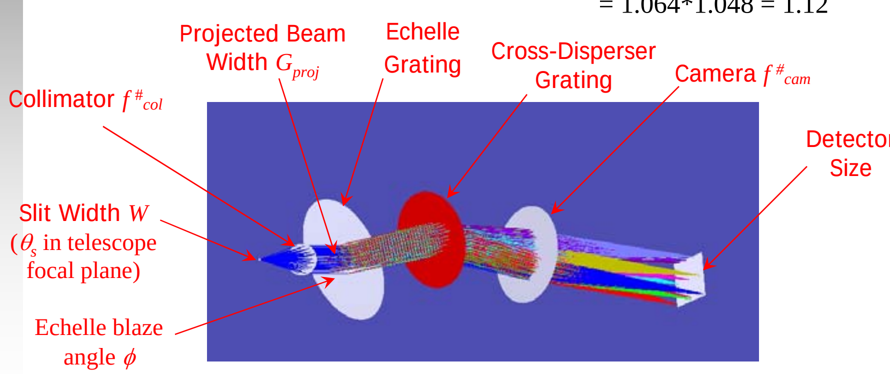
These are not quite the final design specs.

Size of Echelle

- As discussed, the spectral resolution drives the size of the echelle. The figure below shows a schematic layout of a system.

$$G_{proj} = \frac{R}{2\tan\phi}\frac{W}{f^{\#}_{col}} = \frac{R}{2\tan\phi}\frac{\theta_s D_t}{2063} d_{frd}$$

R = $\lambda/\Delta\lambda$
D_t = telescope diameter (in m)
d_{frd} = fiber focal ratio degradation = 1.064*1.048 = 1.12



Governing Equations

- With an IFU, the camera f-number ($f^{\#}_{cam}$), the project size of the echelle (G_{proj}), and the array size ($N_{px} \times N_{py}$) are computed as:

$$f^{\#}_{cam} = \frac{s_{pix}\, p_{samp}}{\theta_s\, D_t} \frac{f_{ifu}}{d_{frd}} \qquad G_{proj} = \frac{R}{2\tan\phi} \frac{\theta_s\, D_t}{f_{ifu}} d_{frd}$$

$$N_{px} N_{py} \approx n_{spo}\, p_{samp} \left(n_{sp} + p_{samp}\, f_{ifu}^2\right) \frac{\lambda_2(\lambda_2 - \lambda_1)}{\lambda_1^2} R$$

s_{pix} = pixel size (= 18 μm)
p_{samp} = sample per resolution element (~3)
f_{ifu} = number of sub-slits (1, 2, 3, …)
D_t = telescope diameter (= 8.1 m)
θ_s = slit size on sky (= 1.1")
d_{frd} = fiber focal ratio degradation = 1.064*1.048 = 1.12

R = λ/Δλ (= 25,000)
ϕ = echelle blaze angle (= 75.96°)
λ_1 = starting λ (= 0.9 μm)
λ_2 = ending λ (= 1.8 μm)
N_{px} = array cols
N_{py} = array rows
n_{spo} = # of spectra per order (= 6)
n_{sp} = gap between spectra (= 2 pix)

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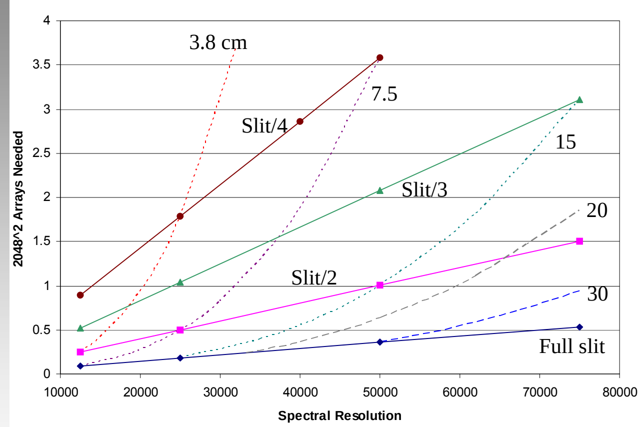
Spectrograph Design Space w/o EDI



Plot of number of required 2048x2048 arrays versus spectral resolution for 8-m telescope with 1.1" slit. Solid lines represent constant slit size. Dashed lines indicate a fixed projected echelle size (in cm) as indicated.

The slit is made smaller by an IFU which divides the slit and remaps it into the cross-dispersion direction.

Assumptions: Full λ-range = 0.9-1.8 μm, 1.1" slit, 1 spectra per order sampled at 3 pixels/resolution element with a spacing of 2 pixels between spectra. The spectrograph size can be reduced by dissecting the slit but at the cost of a larger focal plane.

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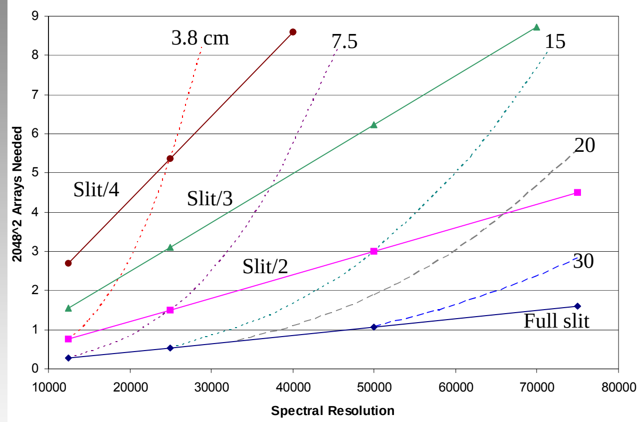
Spectrograph Design Space w/o EDI

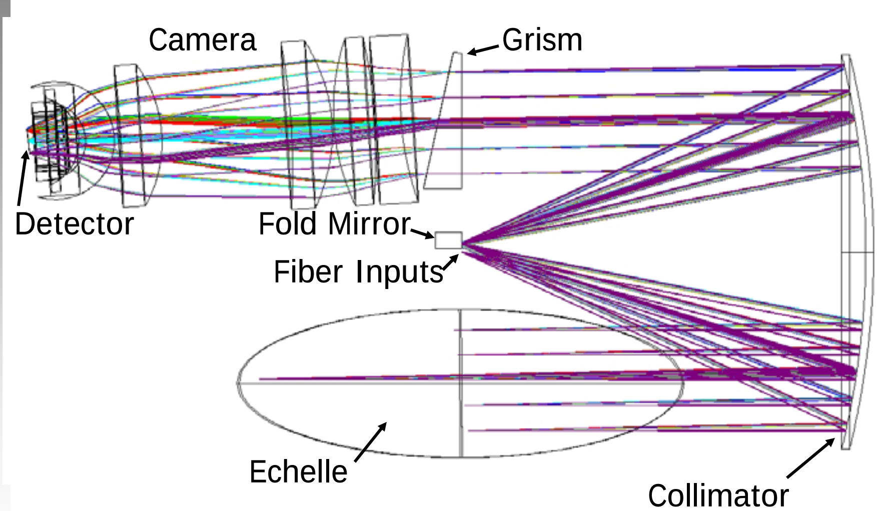


Plot of number of required 2048x2048 arrays versus spectral resolution for 8-m telescope with 1.1" slit. Solid lines represent constant slit size. Dashed lines indicate a fixed projected echelle size (in cm) as indicated.

The slit is made smaller by an IFU which divides the slit and remaps it into the cross-dispersion direction.

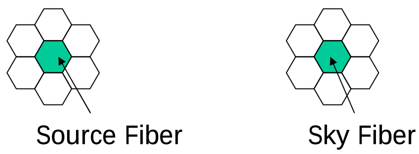
Assumptions: Full λ-range = 0.9-1.8 μm, 1.1" slit, 3 spectra per order sampled at 3 pixels/resolution element with a spacing of 2 pixels between spectra. The spectrograph size can be reduced by dissecting the slit but at the cost of a larger focal plane.

Spectrograph Ray Trace



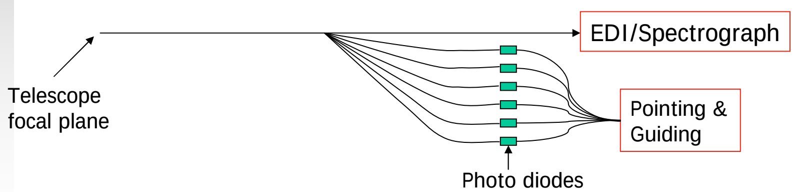
Fiber feed

Layout in telescope focal plane



- Switch between source and sky positions.
- Problem with this design is the we (potentially) get a sqrt(2) noise increase if we subtract a single background fiber.
- Should we have more sky positions (instead of dark areas) on the array? – no room on array.

Fiber path

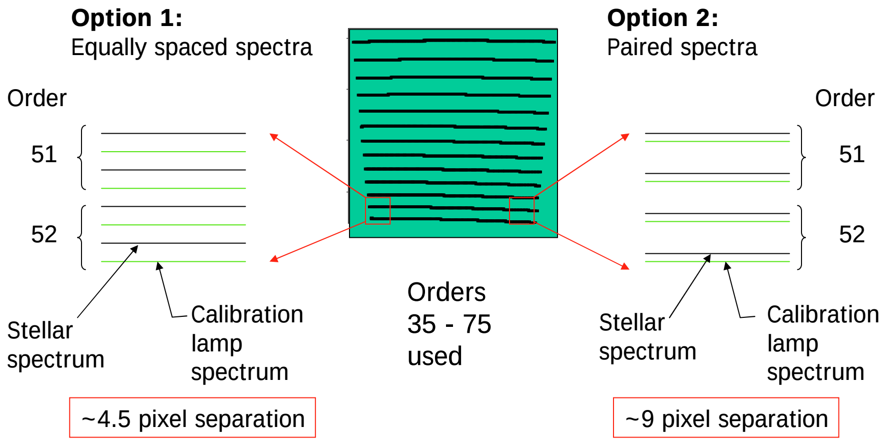


Current Design

R ($\lambda/\Delta\lambda$)	Pix/S E	Num. Orders	Sp/ Order	f/#	P (cm)	Gap (pix)	Array Format
25,000	**3.0**	**35**	**4**	**1.3**	**13.3**	**9**	**2048×2048**
	2.5	29	4	1.1	13.3	14	2048×2048

- Columns are spectral resolution, pixels per resolution element, number of orders, spectra per order, array camera f-number, projected echelle size, gap between spectra, and array format.
- Assumes 1.1" slit. For R = 25,000 the "gap" is computed assuming four spectra per order which are grouped as pairs.

Spectrum layout options



- Accept both output beams from the EDI. Assume each beam has both a stellar spectrum and an emission lamp spectrum
  - Can equally space spectra within an order or
  - Put reference spectrum near stellar spectrum

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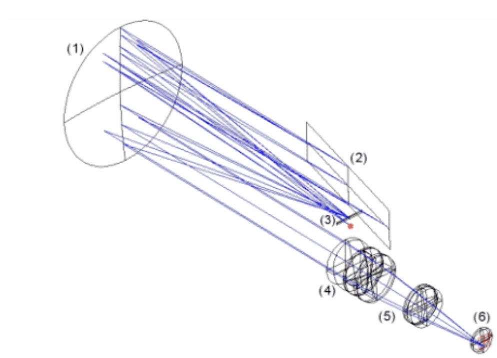
Basic Layout

- Optical path is from entrance point (red dot) to f/7.5 collimator, echelle, collimator (now a camera), fold mirror, collimator, grism, f/1.25 camera, and focal plane with a field lens.

(1) f/7.5 Collimator
(2) Echelle
(3) Fold mirror
(4) Grism
(5) Camera
(6) Focal Plane



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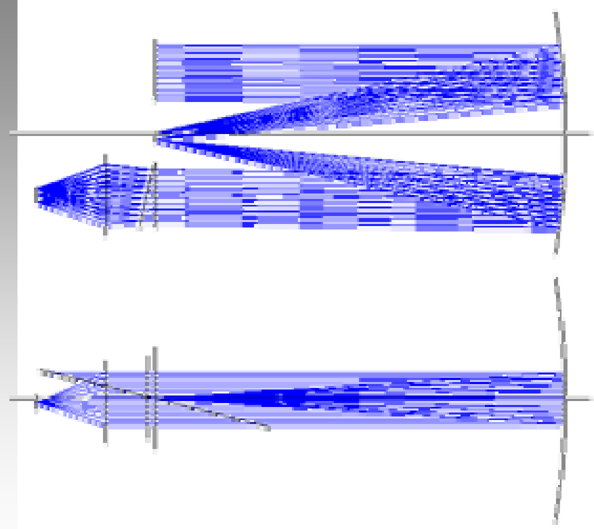
System Ray Trace



View in (y,z) plane

y, z ray trace of system showing optical layout. Echelle is tipped in and out of the paper in this plot.

View in (x,z) plane

Same as above but x,z view. Echelle can be seen in this view. Note that only one "color" is traced, otherwise there would be a range of angles hitting the collimator from the echelle.

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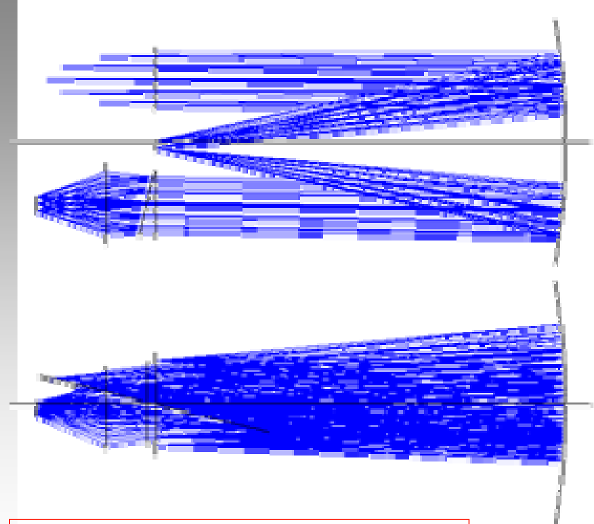
System Ray Trace



View in (y,z) plane

y, z ray trace of system showing optical layout. Echelle is tipped in and out of the paper in this plot.

View in (x,z) plane

Same as above but x,z view. Echelle can be seen in this view. Note that only one "color" is traced, otherwise there would be a range of angles hitting the collimator from the echelle.

Same as previous but all limiting rays

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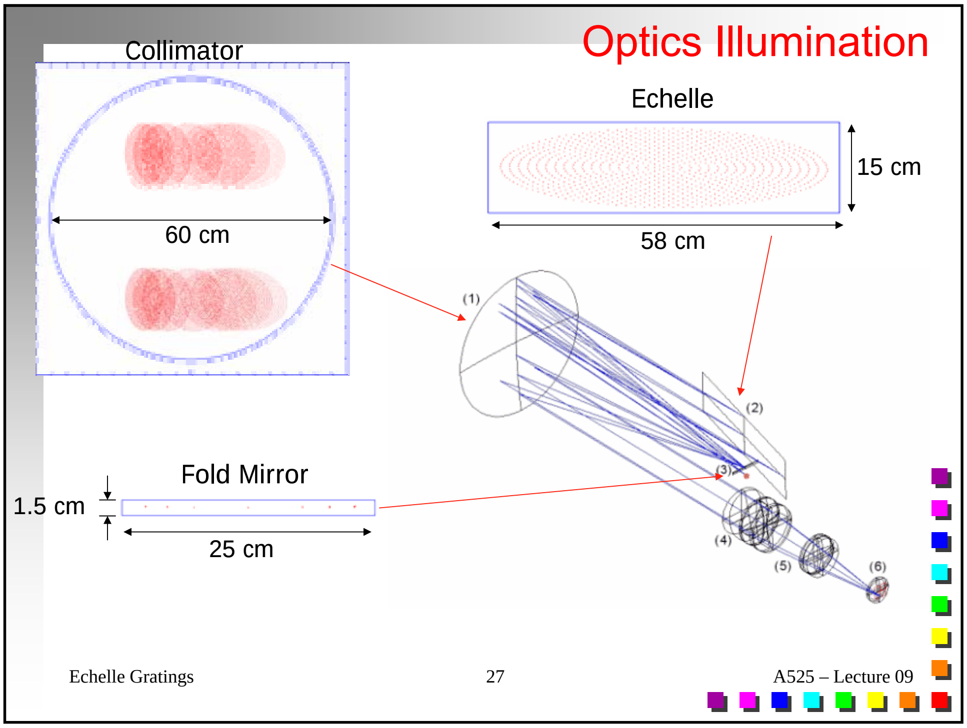
Optics Illumination
Collimator
60 cm
Echelle
15 cm
58 cm
Fold Mirror
1.5 cm
25 cm
(1)
(2)
(3)
(4)
(5)
(6)
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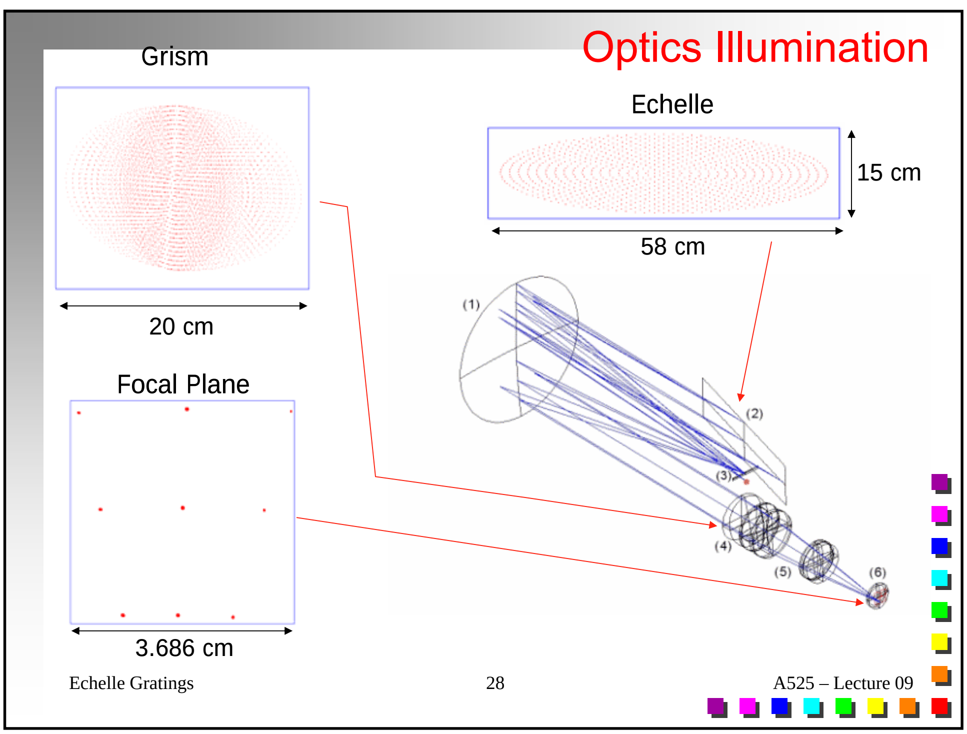
Optics Illumination
Grism
20 cm
Echelle
15 cm
58 cm
Focal Plane
3.686 cm
(1)
(2)
(3)
(4)
(5)
(6)
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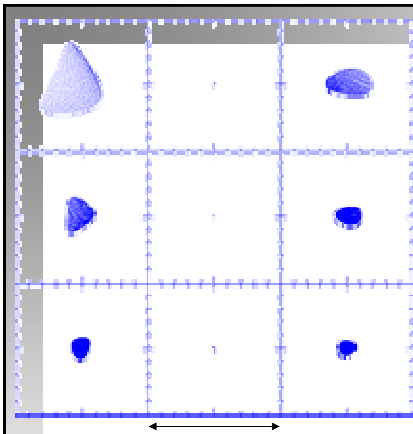
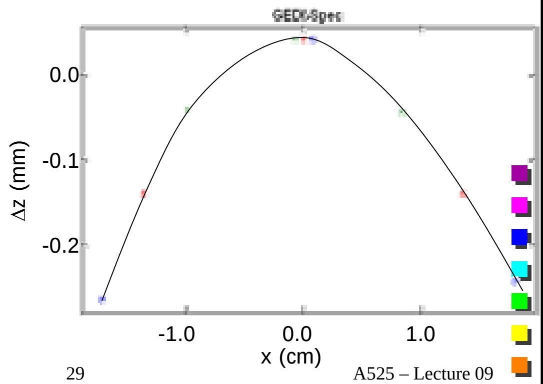
Spot Diagrams



1 pixel

Left: Spot diagrams of extreme colors of system. These have been re-focused to best focus. There is curvature along the echelle dispersion direction (x-axis).

Right: Distance to best focus position from flat focal plane.



GEDI Spectrograph Design Parameters

Echelle:

Blaze Angle:	75.9638 deg [tan(alpha) = 4]	
Groove Spacing:	35 microns in narrow direction	
Size:	58 x 15 cm	
Operation Orders:	38 – 75	
λ Range:	0.9 – 1.8 microns	

Grism:

Apex Angle:	13.0 deg	
Groove Spacing:	4.5 microns in wide direction	
Index of refraction:	2.4	(KRS5)
Size:	20 x 16 cm	
Operation Order:	1	

Collimator:

Focal Length:	100 cm	(f/7.5)
Diameter:	60 cm	
x,y position	0, 0 cm	(relative to slit)

Camera:

Focal Length:	16.875 cm	(f/1.25)
Entrance Diameter:	20 cm	
x,y position	-1, -15.33 cm	(relative to slit)

Image Scale:

Image scale:	1.1 arcsec => 324 micron at entrance to spectrograph
At focal plane:	53.2 microns = 2.96 pixels (18 microns/pixel)

These are not the final design specs, but are illustrative.