Non-ideal Detectors

Astronomy 525

Lecture 22

Outline

- Electrical Bandwidth Review
- Sources of Noise
  - Read Noise
  - Dark Current
  - G-R Noise (photoconductors)
  - *1/f* noise
  - *kTC* noise
- Sensitivity in the real world
  - Combining everything
- Measuring key detector parmeters
  - Read noise, dark current, DQE, RQE

Electrical Bandwidth

- A short reminder about electrical bandwidth
  - From lecture on ideal photon detectors
- If a detection system responds uniformly to modulation frequency between f_1 and f_2 and no response outside:

$$\Delta f = f_2 - f_1$$

- If $R(f)$ varies continuously then

$$\Delta f = \int_0^\infty \left| \frac{R(f)}{R_{\max}} \right|^2 df$$

where $R_{\max} = \max\left(R(f)\right)$

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E-BW: Exponential Decay

- For a system with an exponential decay time, τ,

$$\mathrm{v}(t) = \begin{cases} 0 & t < 0 \\ \mathrm{v}_o\, e^{-t/\tau} & t \geq 0 \end{cases}$$

$$\Rightarrow v(f) = \int_0^\infty \mathrm{v}(t) e^{-i2\pi f t} dt = \frac{\mathrm{v}_o \tau}{1 + i2\pi f \tau}$$

Or

$$\Rightarrow R(f) = \frac{R_o}{1 + i2\pi f \tau}$$

$$\Rightarrow \Delta f = \int_0^\infty \frac{df}{1 + (2\pi f \tau)^2} = \frac{1}{4\tau}$$

DC responsivity is
$R_o = \mathrm{v}_o \tau / Q_o$
$P(t) = Q_o \delta(t)$

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E-BW: Integrator over time T

- For a system that integrates over a time, T, the response is:

$$v(t) = \begin{cases} 1 & -T/2 < t < T/2 \\ 0 & otherwise \end{cases}$$

The frequency response is:

$$\Re(f) = \int_{-T/2}^{T/2} \frac{R_o}{T} e^{-i2\pi f t} dt = R_o \frac{\sin(\pi f T)}{\pi f T}$$

The electrical bandwidth is then

$$\Delta f = \int_0^{\infty} \left[\frac{\sin(\pi f T)}{\pi f T}\right]^2 = \frac{1}{2T}$$

Read Noise

- Read noise: A given a number of (noise) electrons/read
  - Due to whatever source, such as Johnson noise
  - In the ideal case it does not depend upon integration time
- Let R_N = read noise (in units of electrons)

$$N = R_N$$

- The signal is the collected number of electrons in time, *t*

$$S = \frac{\eta P_S}{h\nu} t \qquad \Rightarrow \qquad \frac{S}{N} = \frac{\eta P_S}{h\nu} \frac{t}{R_N}$$

- For Read Noise limited detection

$$NEP = \frac{h\nu}{\eta} \frac{R_N}{t}$$

NEP is P_s for *S/N* = 1

Dark Current

- Dark Current: A background current when no photons are striking the detector.
- Assume it has a shot noise behavior, so that the signal and noise (in electrons) in the dark current after a time, *t* are

$$S_d = i_d t \quad \Rightarrow \quad N_d = \sqrt{i_d t}$$

- So that

$$\frac{S}{N} = \frac{S}{N_d} = \frac{\eta P_S}{h\nu}\sqrt{\frac{t}{i_d}}$$

- For Dark current limited detection

$$NEP = \frac{h\nu}{\eta}\sqrt{\frac{i_d}{t}}$$

Generation-Recombination Noise

- G-R noise in photoconductors
  - Result from the statistical fluctuations in the number of the conduction-band electron (and valence-band holes) that are available to conduct electricity
  - This is because the generation and recombination process may be considered independent stochastic processes
- The photocurrent through a photoconductor will be comprised of pulses which fluctuate randomly.
  - The duration of the pulse is equal to the lifetime of the conduction band electron.

Discussion follows Boyd

Pulse length noise

- The probability of a pulse in the range $(t,t+dt)$ is

$$p(t)dt = \frac{1}{\tau}e^{-t/\tau}dt$$

τ = mean lifetime of an electron in the conduction band

- We now generalize the shot noise analysis performed earlier. The shot noise formula is

$$\overline{i_N^2} = 2e\bar{i}\Delta f$$

- For gr noise the contribution to the noise comes from pulses of length t which gives

$$d\left(\overline{i_N^2}\right) = 2q(t)\Delta f\, d\bar{i}$$

$q(t)$ = charge passing through PC for pulse of length t

$d\bar{i}$ = contribution to $\bar{i}$ resulting from the pulses

Mean-square Current

- Now the charge per pulse is proportional to the length t of the pulse, and since the average value of this charge is Ge,

$$q(t) = \frac{Get}{\tau}$$

- The mean current resulting from the pulses of length t must be proportional to the pulse length, t, and to the probability distribution $p(t)$,

$$d\bar{i} = \bar{i}\,\frac{t}{\tau^2}e^{-t/\tau}dt$$

- Integrating for the mean-square current gives

$$\overline{i_N^2} = 2Ge\bar{i}\Delta f\int_0^\infty \frac{t^2}{\tau^3}e^{-t/\tau}dt = 4Ge\bar{i}\Delta f = 2Ge\bar{i}/t$$

integration time

G-R NEP

- If we now have a large background generating photocurrent, then for a PC

$$\overline{i_N^2} = \frac{2Ge\bar{i}_B}{t} = \frac{2Ge}{t}\frac{\eta GP_B e}{h\nu}$$

- The signal is

$$\bar{i}_S = \frac{\eta GP_S e}{h\nu} \quad \Rightarrow \quad \boxed{NEP = \sqrt{\frac{2P_B h\nu}{\eta t}}}$$

BLIP for photoconductor

- Note If the dark current is present, for a photoconductor the noise expression (in electrons after a time *t*) changes

$$N_d = \sqrt{i_d t} \quad \Rightarrow \quad N_d = \sqrt{2G_d i_d t}$$

1/f Noise

- *1/f* noise: An increase in system noise at low frequencies
- The origin is not well understood, empirically

$$\overline{i_N^2}(f) = \frac{Ki^{\alpha}}{f^{\beta}} \qquad \alpha \simeq 1, \beta \simeq 2$$

- This noise is reduce by "chopping" the signal
  - Modulate the signal at some large frequency
  - Examine only those components in a narrow frequency band that include the modulation frequency
- How this noise enters then depends upon the data taking and the frequency "transfer" function of the system (see homework problem).

kTC Noise

- *kTC* noise: due to inability to set charge to zero upon reset
  - Happens in reset-integrator type circuits
  - The noise voltage is given by (see readings – Barbe 1975)

$$\overline{\mathrm{v}_C(t)^2} = \frac{kT_c}{C}\left(1 - e^{-2t/RC}\right) \xrightarrow[t >> RC]{} \left(\overline{\mathrm{v}_C{}^2}\right)^{1/2} = \sqrt{\frac{kT_c}{C}}$$

- Adds to system read noise (R_{kTC})

$$R_{kTC} \equiv Q_{rms} = CV_{rms} = \sqrt{kT_c C}$$

- Can be “eliminated” by double correlated sampling

Putting it all together

- We now look at estimating the signal-to-noise ratio and sensitivity for a real system
- The most common sources of noise are due to
  - Signal photons
  - Background photons
  - Dark current
  - Read noise
- For the first three above, the noise will be due to shot noise in the collected photoelectrons, that is,

$$Noise \propto \sqrt{\bar{i}\,t}$$

Source Signal

- The number of photoelectrons collected due to the signal will be

$$S = \frac{\eta G \tau \tau_w P_S}{h\nu} t$$

where η = quantum efficiency
τ = transmission of "cold" optics
τ_w = transmission of "warm" optics/atmosphere

- The source signal is $P_S = f_\lambda \Delta\lambda\, A$ — f_λ in ergs/cm²/sec/μm

  or $P_S = I_\lambda \Delta\lambda\, A\Omega$ — I_λ in ergs/cm²/sec/μm/sr

- Where we consider f_λ when dealing with point sources and I_λ for extended sources

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Noise Contributions

- The noise contributions are

$$N_S{}^2 = \beta G S = \frac{\beta G^2 \eta \tau \tau_w P_S}{h\nu} t \qquad \text{Source Signal}$$

$$N_B{}^2 = \beta G S_B = \frac{\beta G^2 \eta \tau P_B}{h\nu} t \qquad \text{Background}$$

$$N_d{}^2 = \beta_d G_d S_d = \beta_d G_d i_d t_d \qquad \text{Dark Current}$$

$$R_N^2 = R_N^2 \qquad \text{Read Noise}$$

where β = gain dispersion (1 for PVs, 2 for PCs)
G = gain (usually photoconductive gain)
β_d, G_d = same but for dark current

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Background Power

- If the background contribution is thermal, then

$$P_B = \varepsilon_\lambda B_\lambda(T) A\Omega \Delta\lambda$$

- Or if multiple sources contribute, we sum over each component
- If the background is non-thermal we can replace $\varepsilon_\lambda B_\lambda(T)$ with $I_{B\lambda}$ where $I_{B\lambda}$ is the intensity of the background, e.g. airglow.
- Note: Ω is the extraction (aperture) size use to obtain the source signal. Since the photon background increases with this, better sensitivity is achieved with smaller apertures (to a point). More on this later.

Signal-to-Noise Ratio

- The signal-to-noise ratio is

$$\frac{S}{N} = \frac{S}{\left[N_S{}^2 + N_B{}^2 + N_d{}^2 + R_N^2\right]^{1/2}}$$

- Since the (uncorrelated) noises add in quadrature
- Thus, solving for S and plugging in gives

$$\frac{\eta G \tau \tau_w P_S}{h\nu} t = \frac{S}{N}\left[\frac{\beta G^2 \eta \tau(\tau_w P_S + P_B)}{h\nu} t + \beta_d G_d \bar{i}_d t + R_N^2\right]^{1/2}$$

- If one of the noise terms dominates the others then the system is called: background, dark current or read noise limited depending on which dominates.
- The best condition is signal-noise-limited

BLIP

- BLIP (Background Limited Infrared Performance) is defined as the background dominating all other noise sources
- Assuming this, we get for the sensitivity

$$f_\lambda = \frac{S}{N}\frac{1}{\tau_w}\sqrt{\frac{h\nu\,\varepsilon_\lambda B_\lambda}{(\eta/\beta)\tau\, t}\frac{\Omega}{\Delta\lambda\, A}}$$

- BLIP is the best you can do (unless your source is bright enough to reach the signal-noise-limited regime)
- Note: G has "disappeared" and the sensitivity depends on the detector parameter η/β (detective quantum efficiency).

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Measuring Noise: Read Noise

- Neglecting contributions from signal photons, the noise is

$$N^2 = \frac{\beta G^2 \eta \tau P_B}{h\nu} t + \beta_d\, G_d \bar{i}_d t + R_N^2$$

- Through adjustment of the integration time and background we can determine each component of the noise.
- This allows us to estimate the performance/sensitivity of an instrument under other conditions
- Read Noise:
  - Make detector dark
  - Choose short integration times $\Rightarrow \quad N^2 = R_N^2$

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Measuring Noise: Dark Current

- Dark Current
  - Make detector dark
  - Make integration time long enough so that $\beta_d G_d \bar{i}_d t >> R_N^2$
- The dark current, i_d can be measured directly from signal vs. time with no light on the detector
- The signal-to-noise ratio on the dark current can be used to determine $\beta_d G_d$

$$\left(\frac{S}{N}\right)^2 = \frac{(\bar{i}_d t)^2}{\beta_d G_d \bar{i}_d t} \quad \Rightarrow \quad \bar{i}_d t \equiv S = \beta_d G_d \left(\frac{S}{N}\right)^2$$

or $\beta_d G_d = \dfrac{N^2}{S}$

Can get from slope of $(S/N)^2$ vs. S

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Detective Quantum Efficiency

- Make P_B and t large enough so that

$$\frac{\beta G^2 \eta \tau P_B}{h\nu} t >> \beta_d G_d \bar{i}_d t \ , \ R_N^2$$

- Now measure S/N on background (background is signal)

$$\frac{S}{N} = \frac{\eta G \tau P_B t / h\nu}{(\beta G^2 \eta \tau P_B t / h\nu)^{1/2}} = \left[\frac{\eta}{\beta}\frac{\tau P_B t}{h\nu}\right]^{1/2} \Rightarrow \left(\frac{S}{N}\right)^2 = \frac{\eta}{\beta}\frac{\tau P_B t}{h\nu}$$

- η/β is call the detective quantum efficiency (DQE)
- You must know photon background on detector
- Note that we get βG from

$$\beta G = N^2 / S$$

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Responsive Quantum Efficiency

- Measure the photocurrent – not the noise

$$S = \eta G \frac{\tau P_B}{h\nu} t$$

- If photon rate at the detector is known, ηG can be measured.
- If G can be estimated, the η can also be determined.
- η determined in this way is called the responsive quantum efficiency

In practice, we can only determine ηG, βG, and η/β but not the individual values.

[For PVs and BIBs we should have $\beta = G = 1$]

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