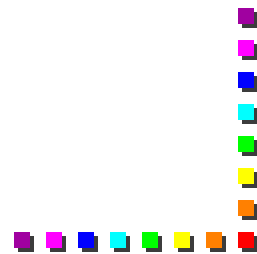


Non-ideal Detectors

Astronomy 525

Lecture 22



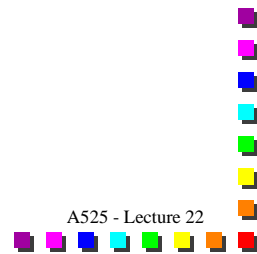
Outline

- Electrical Bandwidth Review
- Sources of Noise
 - Read Noise
 - Dark Current
 - G-R Noise (photoconductors)
 - $1/f$ noise
 - kTC noise
- Sensitivity in the real world
 - Combining everything
- Measuring key detector parameters
 - Read noise, dark current, DQE, RQE

Non-ideal Detectors

2

A525 - Lecture 22



Electrical Bandwidth

- A short reminder about electrical bandwidth
 - From lecture on ideal photon detectors
- If a detection system responds uniformly to modulation frequency between f_1 and f_2 and no response outside:

$$\Delta f = f_2 - f_1$$

- If $R(f)$ varies continuously then

$$\Delta f = \int_0^{\infty} \left| \frac{R(f)}{R_{\max}} \right|^2 df$$

where $R_{\max} = \max(R(f))$

Non-ideal Detectors

3

A525 - Lecture 22

E-BW: Exponential Decay

- For a system with an exponential decay time, τ ,

$$v(t) = \begin{cases} 0 & t < 0 \\ v_o e^{-t/\tau} & t \geq 0 \end{cases}$$

$$\Rightarrow v(f) = \int_0^{\infty} v(t) e^{-i2\pi f t} dt = \frac{v_o \tau}{1 + i2\pi f \tau}$$

Or

$$\Rightarrow R(f) = \frac{R_o}{1 + i2\pi f \tau}$$

DC responsivity is

$$R_o = v_o \tau / Q_o$$

$$P(t) = Q_o \delta(t)$$

$$\Rightarrow \Delta f = \int_0^{\infty} \frac{df}{1 + (2\pi f \tau)^2} = \frac{1}{4\tau}$$

Non-ideal Detectors

4

A525 - Lecture 22

E-BW: Integrator over time T

- For a system that integrates over a time, T, the response is:

$$v(t) = \begin{cases} 1 & -T/2 < t < T/2 \\ 0 & \text{otherwise} \end{cases}$$

The frequency response is:

$$\Re(f) = \int_{-T/2}^{T/2} \frac{R_o}{T} e^{-i2\pi ft} dt = R_o \frac{\sin(\pi fT)}{\pi fT}$$

The electrical bandwidth is then

$$\Delta f = \int_0^{\infty} \left[\frac{\sin(\pi fT)}{\pi fT} \right]^2 = \frac{1}{2T}$$

Non-ideal Detectors

5

A525 - Lecture 22

Read Noise

- Read noise: A given a number of (noise) electrons/read
 - Due to whatever source, such as Johnson noise
 - In the ideal case it does not depend upon integration time
- Let R_N = read noise (in units of electrons)

$$N = R_N$$

- The signal is the collected number of electrons in time, t

$$S = \frac{\eta P_s}{h\nu} t \quad \Rightarrow \quad \frac{S}{N} = \frac{\eta P_s}{h\nu} \frac{t}{R_N}$$

- For **Read Noise limited detection**

$$NEP = \frac{h\nu}{\eta} \frac{R_N}{t}$$

NEP is P_s
for $S/N = 1$

Non-ideal Detectors

6

A525 - Lecture 22

Dark Current

- Dark Current: A background current when no photons are striking the detector.
- Assume it has a shot noise behavior, so that the signal and noise (in electrons) in the dark current after a time, t are

$$S_d = i_d t \quad \Rightarrow \quad N_d = \sqrt{i_d t}$$

- So that $\frac{S}{N} = \frac{S}{N_d} = \frac{\eta P_s}{h\nu} \sqrt{\frac{t}{i_d}}$

- For Dark current limited detection

$$NEP = \frac{h\nu}{\eta} \sqrt{\frac{i_d}{t}}$$

Non-ideal Detectors

7

A525 - Lecture 22

Generation-Recombination Noise

- G-R noise in photoconductors
 - Result from the statistical fluctuations in the number of the conduction-band electron (and valence-band holes) that are available to conduct electricity
 - This is because the generation and recombination process may be considered independent stochastic processes
- The photocurrent through a photoconductor will be comprised of pulses which fluctuate randomly.
 - The duration of the pulse is equal to the lifetime of the conduction band electron.

Discussion follows Boyd

Non-ideal Detectors

8

A525 - Lecture 22

Pulse length noise

- The probability of a pulse in the range $(t, t+dt)$ is

$$p(t)dt = \frac{1}{\tau} e^{-t/\tau} dt \quad \tau = \text{mean lifetime of an electron in the conduction band}$$

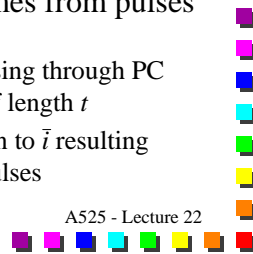
- We now generalize the shot noise analysis performed earlier. The shot noise formula is

$$\overline{i_N^2} = 2e\bar{i}\Delta f$$

- For gr noise the contribution to the noise comes from pulses of length t which gives

$$d(\overline{i_N^2}) = 2q(t)\Delta f d\bar{i}$$

$q(t)$ = charge passing through PC for pulse of length t
 $d\bar{i}$ = contribution to \bar{i} resulting from the pulses



Mean-square Current

- Now the charge per pulse is proportional to the length t of the pulse, and since the average value of this charge is Ge ,

$$q(t) = \frac{Get}{\tau}$$

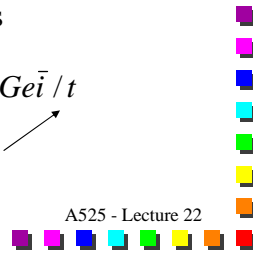
- The mean current resulting from the pulses of length t must be proportional to the pulse length, t , and to the probability distribution $p(t)$,

$$d\bar{i} = \bar{i} \frac{t}{\tau^2} e^{-t/\tau} dt$$

- Integrating for the mean-square current gives

$$\overline{i_N^2} = 2Ge\bar{i}\Delta f \int_0^\infty \frac{t^2}{\tau^3} e^{-t/\tau} dt = 4Ge\bar{i}\Delta f = 2Ge\bar{i} / t$$

integration time \nearrow



G-R NEP

- If we now have a large background generating photocurrent, then for a PC

$$\overline{i_N^2} = \frac{2Ge\overline{i_B}}{t} = \frac{2Ge}{t} \frac{\eta GP_B e}{h\nu}$$

- The signal is

$$\overline{i_s} = \frac{\eta GP_s e}{h\nu} \Rightarrow \boxed{NEP = \sqrt{\frac{2P_B h\nu}{\eta t}}} \quad \text{BLIP for photoconductor}$$

- Note If the dark current is present, for a photoconductor the noise expression (in electrons after a time t) changes

$$N_d = \sqrt{\overline{i_d t}} \Rightarrow N_d = \sqrt{2G_d \overline{i_d t}}$$

Non-ideal Detectors

11

A525 - Lecture 22

$1/f$ Noise

- $1/f$ noise: An increase in system noise at low frequencies
- The origin is not well understood, empirically

$$\overline{i_N^2}(f) = \frac{Ki^\alpha}{f^\beta} \quad \alpha \simeq 1, \beta \simeq 2$$

- This noise is reduce by “chopping” the signal
 - Modulate the signal at some large frequency
 - Examine only those components in a narrow frequency band that include the modulation frequency
- How this noise enters then depends upon the data taking and the frequency “transfer” function of the system (see homework problem).

Non-ideal Detectors

12

A525 - Lecture 22

kTC Noise

- *kTC* noise: due to inability to set charge to zero upon reset
 - Happens in reset-integrator type circuits
 - The noise voltage is given by (see readings – Barbe 1975)

$$\overline{v_c(t)^2} = \frac{kT_c}{C} (1 - e^{-2t/RC}) \xrightarrow{t \gg RC} \left(\overline{v_c^2}\right)^{1/2} = \sqrt{\frac{kT_c}{C}}$$

- Adds to system read noise (R_{kTC})

$$R_{kTC} \equiv Q_{rms} = CV_{rms} = \sqrt{kT_c C}$$

- Can be “eliminated” by double correlated sampling

Putting it all together

- We now look at estimating the signal-to-noise ratio and sensitivity for a real system
- The most common sources of noise are due to
 - Signal photons
 - Background photons
 - Dark current
 - Read noise
- For the first three above, the noise will be due to shot noise in the collected photoelectrons, that is,

$$\text{Noise} \propto \sqrt{it}$$

Source Signal

- The number of photoelectrons collected due to the signal will be

$$S = \frac{\eta G \tau \tau_w P_s t}{h \nu}$$

where η = quantum efficiency
 τ = transmission of “cold” optics
 τ_w = transmission of “warm” optics/atmosphere

- The source signal is $P_s = f_\lambda \Delta \lambda A$ f_λ in ergs/cm²/sec/μm

or

$$P_s = I_\lambda \Delta \lambda A \Omega \quad I_\lambda \text{ in ergs/cm}^2\text{/sec}/\mu\text{m/sr}$$

- Where we consider f_λ when dealing with point sources and I_λ for extended sources



Noise Contributions

- The noise contributions are

$$N_s^2 = \beta G S = \frac{\beta G^2 \eta \tau \tau_w P_s t}{h \nu} \quad \text{Source Signal}$$

$$N_B^2 = \beta G S_B = \frac{\beta G^2 \eta \tau P_B t}{h \nu} \quad \text{Background}$$

$$N_d^2 = \beta_d G_d S_d = \beta_d G_d i_d t_d \quad \text{Dark Current}$$

$$R_N^2 = R_N^2 \quad \text{Read Noise}$$

where β = gain dispersion (1 for PVs, 2 for PCs)
 G = gain (usually photoconductive gain)
 β_d, G_d = same but for dark current



Background Power

- If the background contribution is thermal, then

$$P_B = \varepsilon_\lambda B_\lambda(T) A \Omega \Delta\lambda$$

- Or if multiple sources contribute, we sum over each component
- If the background is non-thermal we can replace $\varepsilon_\lambda B_\lambda(T)$ with $I_{B\lambda}$ where $I_{B\lambda}$ is the intensity of the background, e.g. airglow.
- Note: Ω is the extraction (aperture) size use to obtain the source signal. Since the photon background increases with this, better sensitivity is achieved with smaller apertures (to a point). More on this later.

Signal-to-Noise Ratio

- The signal-to-noise ratio is

$$\frac{S}{N} = \frac{S}{\left[N_S^2 + N_B^2 + N_d^2 + R_N^2 \right]^{1/2}}$$

- Since the (uncorrelated) noises add in quadrature
- Thus, solving for S and plugging in gives

$$\frac{\eta G \tau \tau_w P_S}{h \nu} t = \frac{S}{N} \left[\frac{\beta G^2 \eta \tau (\tau_w P_S + P_B)}{h \nu} t + \beta_d G_d \bar{i}_d t + R_N^2 \right]^{1/2}$$

- If one of the noise terms dominates the others then the system is called: **background, dark current or read noise limited** depending on which dominates.
- The best condition is **signal-noise-limited**

BLIP

- BLIP (Background Limited Infrared Performance) is defined as the background dominating all other noise sources
- Assuming this, we get for the sensitivity

$$f_{\lambda} = \frac{S}{N} \frac{1}{\tau_w} \sqrt{\frac{h\nu \varepsilon_{\lambda} B_{\lambda}}{(\eta/\beta)\tau t} \frac{\Omega}{\Delta\lambda A}}$$

- BLIP is the best you can do (unless your source is bright enough to reach the signal-noise-limited regime)
- Note: G has “disappeared” and the sensitivity depends on the detector parameter η/β (detective quantum efficiency).

Measuring Noise: Read Noise

- Neglecting contributions from signal photons, the noise is

$$N^2 = \frac{\beta G^2 \eta \tau P_B}{h\nu} t + \beta_d G_d \bar{i}_d t + R_N^2$$

- Through adjustment of the integration time and background we can determine each component of the noise.
- This allows us to estimate the performance/sensitivity of an instrument under other conditions
- **Read Noise:**
 - Make detector dark
 - Choose short integration times $\Rightarrow N^2 = R_N^2$

Measuring Noise: Dark Current

- Dark Current
 - Make detector dark
 - Make integration time long enough so that $\beta_d G_d \bar{i}_d t \gg R_N^2$
- The dark current, i_d can be measured directly from signal vs. time with no light on the detector
- The **signal-to-noise ratio on the dark current** can be used to determine $\beta_d G_d$

$$\left(\frac{S}{N}\right)^2 = \frac{(\bar{i}_d t)^2}{\beta_d G_d \bar{i}_d t} \Rightarrow \bar{i}_d t \equiv S = \beta_d G_d \left(\frac{S}{N}\right)^2$$

\uparrow
 Can get from slope of $(S/N)^2$ vs. S

$$\text{or } \beta_d G_d = \frac{N^2}{S}$$



Detective Quantum Efficiency

- Make P_B and t large enough so that

$$\frac{\beta G^2 \eta \tau P_B t}{h\nu} \gg \beta_d G_d \bar{i}_d t, R_N^2$$

- Now measure S/N on background (background is signal)

$$\frac{S}{N} = \frac{\eta G \tau P_B t / h\nu}{(\beta G^2 \eta \tau P_B t / h\nu)^{1/2}} = \left[\frac{\eta \tau P_B t}{\beta h\nu} \right]^{1/2} \Rightarrow \left(\frac{S}{N}\right)^2 = \frac{\eta \tau P_B t}{\beta h\nu}$$

- η/β is call the detective quantum efficiency (DQE)
- You must know photon background on detector
- Note that we get βG from

$$\beta G = N^2 / S$$



Responsive Quantum Efficiency

- Measure the photocurrent – not the noise

$$S = \eta G \frac{\tau P_B}{h\nu} t$$

- If photon rate at the detector is known, ηG can be measured.
- If G can be estimated, the η can also be determined.
- η determined in this way is called the **responsive quantum efficiency**

In practice, we can only determine ηG , βG , and η/β but not the individual values.

[For PVs and BIBs we should have $\beta = G = 1$]

