Bolometers: I

Astronomy 525

Lecture 23

Outline

- Simple theory
- Simple bolometer
- Electrical properties
- Biasing and read-out circuit
- Time response
- Responsivity
- Noise equivalent power
  - Johnson noise
  - Thermal noise
  - Photon noise

Simple Theory I

- A bolometer is a device that heats up by absorbing radiation. The change in temperature is sensed in some way resulting in the signal.
- Unlike other detectors, bolometers ***do not*** detect photons by direct excitation of charge carriers.
- Photons absorbed ⇒ heat device ⇒ detected with thermometer – a simple example is a mercury thermometer.

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Simple Theory II

- Frequency is irrelevant, absorbed power is the issue ⇒ broad wavelength response
- Absorber can be decoupled from detection ⇒ Very high (approaching 100%) quantum efficiencies are possible.
- Semiconductor bolometers are thermal detectors of choice for IR/submm light but…
  - Bolometers must be operated at extremely low temperatures, typically T < 1 K ⇒ devices like ^{3}He refrigerators, or adiabatic demagnetization refrigerators are used
  - Thermal isolation can be a problem at very low temperatures
  - Broad wavelength coverage can also be a *disadvantage* 2 K radiation from within the instrument can be a problem!
  - 2-d arrays are challenging, but great progress has recently been made.

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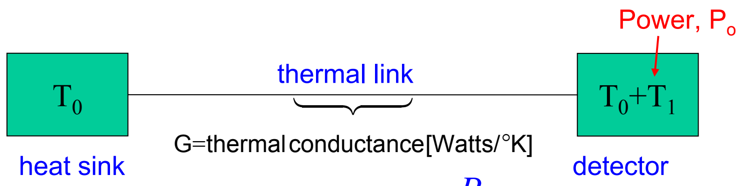
Simple Theory III



The definition of G is such that: $G = \dfrac{P_0}{T_1}$

Now, introduce $P_\nu(t)$ $\begin{cases} \text{time variable} \\ \text{deposited by radiation} \end{cases}$

The detector temperature will therefore change so that: $\eta P_\nu(t) = \dfrac{dQ}{dt} = C \dfrac{dT_1}{dt}$

quantum efficiency

heat capacity [Joules/K] ($dQ = CdT_1$)

Simple Theory IV

The total power the detector absorbs is then:

$$P_T(t) = P_0 + \eta P_\nu(t) = GT_1 + C\frac{dT_1}{dt}$$

Suppose $P_\nu(t) = 0$ for $t < 0$ and $P_\nu(t) = P_1$ for $t \geq 0$. Then

$$T_1(t) = \begin{cases} P_o/G & t < 0 \\ P_o/G + (\eta P_1/G)\{1 - \exp(-t/(C/G)\} & t \geq 0 \end{cases}$$

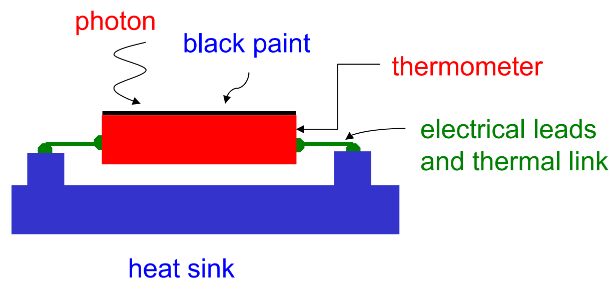
So that the thermal time constant is: $\tau_T = \dfrac{C}{G}$

heat capacity

thermal conductance

For $t >> \tau_T$, $T_1 \propto (P_0 + \eta P_1)$ so that *measuring T_1 measures the input power.*

Simple Bolometer



- Thermometer is a small piece of silicon or germanium doped for high resistance and large temperature coefficient of resistance: $R(T) \sim T^x$
- Measure $V(t)$ across bolometer $\Leftrightarrow R(t) \Leftrightarrow T_1(t) \Leftrightarrow P_1(t)$
- Black paint enhances absorption

Electrical Properties I

- The performance of a bolometer can be characterized by the temperature dependence of its electrical properties.
- This dependence described by: temperature coefficient of resistance:

$$\alpha(t) = \frac{1}{R}\frac{dR}{dT} \ [\mathrm{K}^{-1}]$$

- Assuming electron mobility is independent of temperature, for semiconductors it can be shown that:

$$R = R_0 T^{-3/2} e^{B/T}$$

[$\propto 1/n_o$ where n_o is the number of e^{-1} in the conduction band]

- This formalism is appropriate for modest performance bolometers operating at temperature well above 0 – e.g. near room temperature “thermistor bolometers” made of diamond or germanium.

Electrical Properties II

- High performance bolometers require very low operating T
- To obtain appropriate electrical properties when T < 5 K we need heavy doping leading to hopping conductivity (electrons can jump from one impurity to the next w/o entering the conduction band.) This mechanism freezes out slowly:

$$R = R_0\, e^{(\Delta/T)^{\xi}}, \quad \xi = \tfrac{1}{2}$$

Only valid for T << Δ, where Δ = 4 → 10K

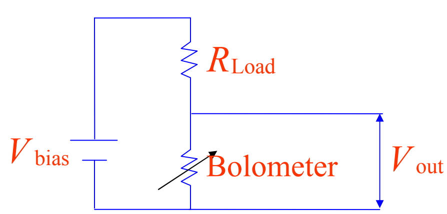
$$\Rightarrow \alpha(T) = -\frac{1}{2}\left[\frac{\Delta}{T^3}\right]^{1/2}$$

- At intermediate temperatures, a good empirical fit is given by:

$$R = R_0\left(\frac{T}{T_0}\right)^{-A} \qquad A \approx 4 \Rightarrow \qquad \alpha(T) = \frac{-A}{T}$$

- Notice that for both of these cases, $\alpha < 0$, and has a strong T dependence *(good!).*

Biasing & readout circuit



- For thermistor sensed bolometers, the simple circuit above is adequate for sensing the temperature changes in the bolometer.
- We analyze this circuit by assuming $R_{load} >> R_{bolometer} \Rightarrow$ to first order, the current through R_{bolo} is independent of R_{bolo}
- Important! If R_{load} is too small $\Rightarrow R_{bolo}$ controls the current, then additional current would heat the bolometer ⇒ R_{bolo} goes down ⇒ more current ⇒ puff!
- For protection, a large R_{load} is nearly always used

Time Response: I

Let $P_I = I^2 R(T)$ be the electrical power dissipated in the detector resistance, $R(T)$ by the current I. $R(T)$ is a function of T so that:

$$P_T(t) = \underbrace{C\frac{dT_1}{dt}}_{\eta \cdot P_\nu(t)} + \underbrace{GT_1}_{P_0} - \underbrace{\frac{dP_I}{dT}T_1}_{\text{system responding to } R \text{ being } R(T)}$$

Notice, that since I = constant, we have:

$$\frac{d\,P_I}{dT} = I^2\frac{dR}{dT} = \alpha\, I^2 R = \alpha\, P_I \qquad \alpha(t) = \frac{1}{R}\frac{dR}{dT}\ [\mathrm{K}^{-1}]$$

so that, combining the above and rearranging:

$$P_T(t) = (G - \alpha\, P_I)T_1 + C\frac{d\,T_1}{dt} \qquad \text{Equation 1}$$

Time Response: II

This again has a solution with an exponential time response, but now $\tau_{\text{electrical}}$ is given by:

$$\tau_E = \frac{C}{G - \alpha\, P_I}$$

- Since the T coefficient of resistance, $\alpha < 0$,

the electrical time constant for a semi-conductor bolometer is shorter than its thermal time constant

$$\tau_T = \frac{C}{G}$$

- This result is called to electro-thermal feedback, and is very *important -- it makes bolometers faster & reduces Johnson noise.*
- Because the time response is exponential, the response as a function of frequency is given by:

$$S(f) = \frac{S(0)}{\left[1 + (2\pi f \tau_E)^2\right]^{1/2}}$$

Low frequency responsivity

[Volts/Watt]

Output signal input photon power

Responsivity: I

recall I is a constant

Let dR, dT, & dV be the changes in R, T & V across the bolometer when there is a change in absorbed power, dP. Then

$$dV = IdR = I(\alpha R dT) = \alpha V dT \qquad \text{Equation 2}$$

Equation 1, above has the solution:

$$T_1(t) = \frac{P_0}{G - \alpha P_I} + \frac{P_I}{G - \alpha P_I}\left[1 - \exp\left(-t/(C/(G - \alpha P_I))\right)\right]$$

So that when $t >> \tau_E = \dfrac{C}{G - \alpha P_I}$ $\quad dT = \dfrac{dP}{G - \alpha P_I}$ $\Rightarrow$ (with Eq. 2)

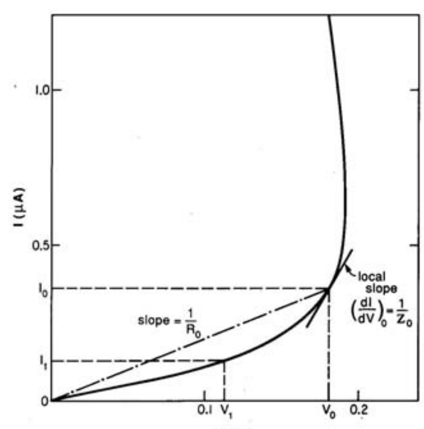
$$dV = \alpha V dT = \frac{\alpha V dP}{G - \alpha P_I} \Rightarrow \boxed{S_E = \frac{dV}{dP} = \frac{\alpha V}{G - \alpha P_I} \quad \text{[volts/Watt]}}$$

This is the electrical responsivity.

Responsivity: II

The temperature coefficient of resistance, α, and the thermal conductance, G, are often not available. How do we characterize our bolometer?

Measure the voltage across the detector as a function of the current through it – can then determine performance parameters. In practice, adjust bias, & measure $V_{detector}$.



Let's look at a typical point, I_0, V_0.
Here $R_0 = V_0/I_0$ and the slope of the I-V curve is $1/Z$:

$$Z = dV/dI$$

(not = R since detector is non-linear)

Responsivity: III

We can rewrite Z as:

$$Z = R\frac{d(\log V)}{d(\log I)} = R\frac{\left[\frac{d(\log P)}{d(\log R)}+1\right]}{\left[\frac{d(\log P)}{d(\log R)}-1\right]}$$ Equation 3

Notice that:

$$\frac{d(\log P)}{d(\log R)} = \frac{R}{P}\frac{dP}{dR} = \frac{1}{I^2}\frac{dP}{dR} = \frac{1}{I^2}\frac{dP}{dT}\frac{dT}{dR}$$

$$= \frac{1}{I^2}G\frac{1}{R\,\alpha} \qquad [\text{since } G = \frac{P}{T}; \alpha = \frac{1}{R}\frac{dR}{dT}]$$

$$= \frac{G}{\alpha P} \equiv H$$

So that we can substitute this into Eq. 3 above, and solve for H:

$$\boxed{H = \frac{Z+R}{Z-R}}$$

determined from the load curve

Responsivity: IV

From the definition of H, then the electrical responsivity is given by:

$$S_E = \frac{\alpha V}{G-\alpha P_I} = \frac{V}{P_I}\frac{1}{\left(\frac{G}{\alpha P_I}-1\right)} = \frac{V}{P_I(H-1)} = \frac{1}{2I}\left(\frac{Z}{R}-1\right)$$

- P_I is the electrical power dissipated in the detector at the operating point on the I-V curve, and $Z, I,$ and R are measured here as well.
- We can also solve for $\tau_T = C/G$:

$$\tau_E = \frac{C}{G-\alpha P} = \frac{\tau_T}{1-\alpha P/G} = \frac{\tau_T}{1-1/H} = \tau_T\left[\frac{Z+R}{2R}\right]\times\left(\frac{R_L+R}{R_L+Z}\right)$$

Correction term for finite R_L (Mather 1982)

- τ_E is easily obtained by looking at the bolometer response to a time varying signal. ⇒ get $\tau_T = C/G$: [G = P_0/T_1]
- We convert S_E to the responsivity to radiation via η, and note that:

$$S_R = \frac{\eta}{2I}\left(\frac{Z}{R}-1\right)$$

S_R is independent of wavelength! (recall for photoconductors, S is proportional to λ and η)

Noise Equivalent Power (NEP)

Three fundamental sources of noise in a bolometer:

(1) Johnson noise, (2) Thermal noise, (3) Photon noise

1. Johnson noise arises from to thermal motions of charge carriers in any resistive circuit element. In a bolometer, we must also include the electro-thermal feedback mechanism:
   --when $V_J = \langle I_J^2 \rangle^{\frac{1}{2}} R$ is added to the bias voltage, the power dissipated increases $\Rightarrow$ heating $\Rightarrow$ *lower resistance* $\Rightarrow$ lowered V_J across the detector $\Rightarrow$ *lower Johnson noise.*
   --Similarly when V_J opposes the bias, the power dissipated decreases $\Rightarrow$ *cooling* $\Rightarrow$ higher resistance, and lower current decreasing the net voltage change across the detector $\Rightarrow$ *lower Johnson noise.*

Johnson Noise: I

The detector response acts to oppose the Ohmic voltage changes from Johnson noise $\Rightarrow$ observed noise is less than:

$$V_J = \langle I_J^2 \rangle^{\frac{1}{2}} R; \qquad \langle I_J^2 \rangle = 4kT\frac{df}{R}$$

If we ignore the changes in operating conditions produced by Johnson noise voltage, the analysis is easier. This means assuming

$$I >> \langle I_J^2 \rangle^{1/2} \text{ and } R_L >> R_D$$

If the bolometer had no signal or noise, then $V_0 = IR + \underbrace{I^2 R S_E}_{\text{response to power dissipated by bias current}}$
adding Johnson noise we have:

$$V_0 + \langle V_N^2 \rangle^{1/2} = \left(IR + \langle I_J^2 \rangle^{1/2}\right)R + \left(I + \langle I_J^2 \rangle^{1/2}\right)^2 R S_E$$

$$\approx const. + \langle I_J^2 \rangle^{1/2} R + 2I\langle I_J^2 \rangle^{1/2} R S_E$$

Johnson Noise: II

The noise is the variable component of this, so that using our expression for S_E:

$$S_E = \frac{1}{2I}\left(\frac{Z}{R} - 1\right):$$

$$\left\langle V_N^2 \right\rangle^{1/2} \approx \left\langle I_J^2 \right\rangle^{1/2} R + \frac{\left\langle I_J^2 \right\rangle^{1/2} R}{2}\left(\frac{Z}{R} - 1\right)$$

$$\approx \left(\frac{R+Z}{2}\right)\left(\frac{4kTdf}{R}\right)^{1/2}$$

We divided the second expression by 2 for the bandwidth response of the bolometer

Notice that if $Z = 0$, the noise is reduced by electro-feedback by a factor of 2 relative to Johnson noise for a fixed R. For a small bias voltage, $Z \approx R$ & nearly the full Johnson noise is expressed.

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Johnson Noise: III

Under Johnson noise limited operation, then, the NEP is:

$$NEP_J = \frac{\left\langle V_N^2 \right\rangle^{1/2}}{S_R\left(df\right)^{1/2}} = \left(4kTR\right)^{1/2} \frac{I}{\eta}\left|\frac{Z+R}{Z-R}\right| \quad \left[W / Hz^{1/2}\right]$$

$$= \left(4kTP\right)^{1/2}\frac{|H|}{\eta} = \left(\frac{4kT}{P}\right)^{1/2}\frac{G}{\eta\left|\alpha(T)\right|} \quad \left[\mathrm{H} \equiv \frac{\mathrm{G}}{\alpha \mathrm{P}} = \frac{Z+R}{Z-R}\right]$$

We have ignored the frequency dependence of NEP_J (multiply by $1/\left[1 + (2\pi f\, \tau_E)^2\right]^{1/2}$ to include). If we substitute in:

low T $\quad \alpha(T) = \frac{-1}{2}\left(\frac{\Delta}{T^3}\right)^{1/2}$

intermediate T $\quad \alpha(T) = \frac{-A}{T}$

$$\Rightarrow \quad NEP_J = \begin{cases} G\,T^2 \\ G\,T^{3/2} \end{cases}$$

It is good to operate at very low temperatures! The T dependence of G can help this further still.

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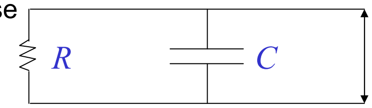
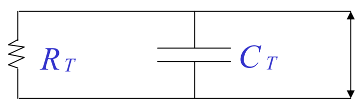
Thermal Noise: I

2. Thermal noise is due to fluctuations in entropy across the thermal link to the heat sink – Detailed derivation is in Mather 1982 -- here we derive an expression by analogy with Johnson noise



Johnson noise circuit



Thermal noise circuit

The thermal link is replaced by an equivalent resistance R_T, and the ability of the bolometer to store energy is represented by C_T.
Energy stored fluctuates under thermodynamic equilibrium:

$$T_N \left(\Leftrightarrow V_N \right)$$

Thermal Noise: II

By analogue:

$$E_T = \underbrace{\frac{1}{2} C_T T_N^2}_{\frac{1}{2} \mathrm{C} \cdot \mathrm{V}_\mathrm{N}^2}$$

Thermal capacitance $[J/K^2]$

$\frac{1}{2} \mathrm{C} \cdot \mathrm{V}_\mathrm{N}^2$ ←energy stored in a capacitor

$$P_T = \frac{T_N^2}{R_T}$$

$[\mathrm{K}^2/\mathrm{W}]$

$I^2 R = \frac{V^2}{R}$ ←rate at which energy is dissipated in R

Thermal Noise: III

Also by analogue, the circuit has an exponential time response with time constant: $\tau_T = \underbrace{R_T C_T}_{\text{heat capacity}}$ But, since $\tau_T = C/G$ we have:

$$R_T C_T = C/G \Rightarrow \underbrace{C_T}_{\substack{\text{thermal}\\ \text{capacitance}}} = \overbrace{C/T}^{\text{heat capacity}} \quad [\mathrm{J/K^2}] \text{ and } R_T = T/G$$

Continuing the analogue:

$$\langle V_J^2 \rangle = (4kTR)df \Rightarrow \langle T_N^2 \rangle = \frac{4kT^2 df}{G}$$

since: $\langle T_N^2 \rangle \Leftrightarrow \langle V_J^2 \rangle$, $R_T \Leftrightarrow R$, $R_T = \frac{T}{G}$

Let a variable signal power P_ν, fall on the bolometer.

Then:

$$\Delta T_S = \frac{\eta P_\nu}{G}$$

Thermal Noise: IV

Letting $\Delta T_S = \langle T_N^2 \rangle^{1/2}$ we have the value of P_ν that produces unit SNR against thermal fluctuations:

$$P_\nu = \frac{(4kT^2 G df)^{1/2}}{\eta}$$

and, hence

$$\downarrow 1\,\mathrm{Hz}$$

$$NEP_T = \frac{(4kT^2 G)^{1/2}}{\eta}$$

Again, clearly operating at low T is good…

Photon Noise

3) Photon noise is noise inherent in the statistical arrival rates of detected photons (∝√N if uncorrelated).

Photon arrival rate (s^{-1})

Bolometers do not have generation-recombination noise so that:

$$NEP_{ph} = \frac{hc}{\lambda}\left(\frac{2\phi}{\eta}\right)^{1/2},$$

Factor of 2 from df =1Hz, df=1/($2\Delta t_{int}$)

The net NEP is the quadratic sum of the constituents:

$$NEP = \left(NEP_J^2 + NEP_T^2 + NEP_{ph}^2 + \cdots\right)^{1/2}$$

The NEP will normally be a function of frequency of operation:

$$S(f) = \frac{S(0)}{\left[1+\left(2\pi \cdot f\,\tau_e\right)^2\right]^{1/2}}$$