Heterodyne Detection: I

Astronomy 525

Lecture 30

Outline

Heterodyne Detection: I

- Photo-detective Devices
- Basic Principles of Coherent Detection
- The “Mixer”: Visible and Infrared
- Conversion Gain
- Fundamental Limits on Noise
- Noise Equivalent Power

Photo-detective Devices

Photon Detectors: Respond to individual photons: photon releases bound charge carriers.

Thermal Detectors: Absorb photons & thermalize their energy: result is a change in the electrical properties of the detector.

Coherent Detectors: Respond to the electric field of the incoming photons

Basic Principles of Coherent Receivers: I

Basic Principle: Incoming photons "mixed" with a local oscillating field to produce a signal at the difference, or *beat frequency*

Encodes the spectrum of the incoming signal at this lower frequency $\Rightarrow$ retains phase information

$\Rightarrow$ readily adapted to spectroscopy

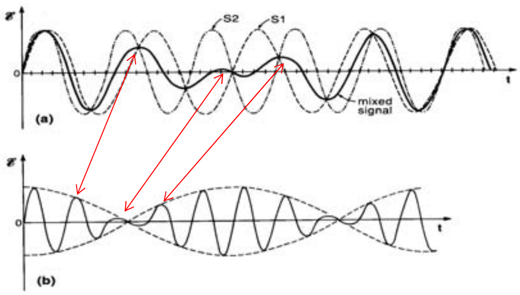
Incoming wave-front can be reconstructed

$\Rightarrow$ readily adaptable to interferometry

Coherent receivers are the instruments of choice at λ's > about a mm, but are rare at shorter wavelengths:

- narrow spectral bandwidths
- array manufacture very difficult – both devices and backends
- small $A\Omega$
- - prohibitive quantum noise

Basic Principles of Coherent Receivers: II



ω_1, ω_2

$\omega_{diff} = \omega_1 - \omega_2$

Heterodyne receivers mix signals (electric fields) of different frequency that "beat" against one another

The resulting field contains frequencies from the original frequencies, but the amplitude is modulated at the difference frequency.

Heterodyne receivers measure the amplitude of the difference frequency

Basic Principles of Coherent Receivers: III

Suppose the resulting field is measured with a *linear device:*

1) If device is fast enough, it simply follows the solid curve containing power only at ω_1 and ω_2

$\Rightarrow$ *no down conversion*

2) If device is slow, there is no signal, since on average the electric field, $\varepsilon = 0$

$\Rightarrow$ *no down conversion*

For heterodyne operation, we need a "mixer" that converts power at ω_1 and ω_2 to $\omega_1 - \omega_2$ *(conversion)*

A non-linear circuit element can do the trick.

- visible and infrared: photon detector, or photomixer
- submm and radio: diode or junction

Basic Principles of Coherent Receivers: IV

For a linear element (a) conversion is zero.

Similarly, any odd function (b) has zero conversion if operated at zero bias - however, conversion can occur if operated with non-zero bias A, since the change in current is larger for + voltage swings than for - voltage swings $\Rightarrow$ a net signal at ω_1-ω_2 i.e. conversion.

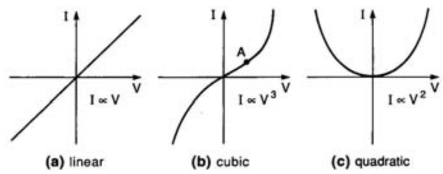


Figure 10.2. I–V curves of three hypothetical mixer elements.

Much greater conversion efficiency is obtained for even function I-V curves (c). Especially nice is a square law detector: $I \propto V^2$. Then, since $V \propto E$, we have: $I \propto V^2 \propto |E|^2 \propto$ Power

The square law detector output is linear with input power!

Basic Principles of Coherent Receivers: V

This is an ideal mixer. In practice, we can assume the I-V curve for any mixer is approximated by a Taylor series:

$$I(V) = I(V_o) + \left(\frac{dI}{dV}\right)_{V=V_0} dV + \frac{1}{2!}\left(\frac{d^2I}{dV^2}\right)_{V=V_0} dV^2 + \frac{1}{3!}\left(\frac{d^3I}{dV^3}\right)_{V=V_0} dV^3 + \ldots$$

- $I(V_0)$ D.C. Current - net response = 0 X
- $\left(\frac{dI}{dV}\right)_{V=V_0} dV$ Linear term - net response = 0 X
- Good square law detector $\Rightarrow$ $\frac{d^2I}{dV^2}$ LARGE
- Higher order terms can be ignored if $dV \equiv (V-V_0)$ is small

Basic Principles of Coherent Receivers: VI

What is happening? Square law detector illuminated by:

ω_{source}: signal frequency

ω_{LO} : local oscillator frequency

Assume: $P_{LO} >> P_{source}$; $\omega_s > \omega_{LO}$

Mixed Signal: $\omega_{IF} \equiv |\omega_{source} - \omega_{LO}|$ -- the intermediate frequency

down converted - easy to process

Yields: Amplitude modulated signal at ω_{IF}

ω_{IF} contains spectral & phase information of ω_s

- ω_{LO} must be stable in ω & phase — laser
- $\omega_{IF} < \omega_{max}$ for equipment — fast

Heterodyne: 1 9 A525 – Lecture 30

Basic Principles of Coherent Receivers: VII

No way of telling in ω_{IF} if $\omega_{source} > \omega_{LO}$ or $\omega_{source} < \omega_{LO}$

- double sideband signal

- no problem for continuum
- can be a big problem for lines

Possible to suppress "image sideband" with filter in front of receiver e.g. Fabry-Perot

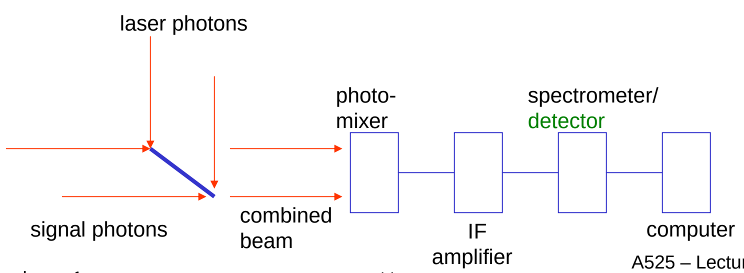
Output from mixer goes to IF amplifier then to spectrometer/detector

- Filter banks
- Correlators
- Acousto-optical spectrometer (AOS)

Heterodyne: 1 10 A525 – Lecture 30

The Mixer: Visible and Infrared I

- At high frequencies, a continuous wave (CW) laser is used as the local oscillator
- Beams are combined by a beam splitter: diplexer
- Output is mixed in the photon detector: photoconductor, photodiode, photomultiplier, bolometer ⇒ responds to power $\propto |\varepsilon|^2$ ⇒ square law mixer



The Mixer: Visible and Infrared II

- Mixer needs to respond to IF frequencies, in the GHz range

  ⇒ careful optimization for high frequency response

  How it works: Assume LO photon beam and source photon beams are perfectly mixed and have the same polarization. Then:

$$E_T(t) = E_L(t) + E_S(t)$$

total electric field

LO $\vec{E}$

source $\vec{E}$

For any EM wave we may write:

$$E(t) = E_0 e^{-i\omega t} = E_0 \cos \omega t - iE_0 \sin \omega t$$
$$H(t) = H_0 e^{-i\omega t} = H_0 \cos \omega t - iH_0 \sin \omega t$$

The Mixer: Visible and Infrared III

Recall the complex amplitudes of the electric, and magnetic fields (ε_0 & H_0) are related by:

$$E_0 = (\frac{\mu}{\varepsilon})^{1/2} H_0$$

where μ is the magnetic permeability, and ε is the dielectric permittivity of the medium

The rate at which energy passes through unit area normal to the wave propagation direction is:

$$S(t) = \mathrm{Re}[E(t)]\mathrm{Re}[H(t)]\hat{i}$$

real part of Poynting vector

direction of S

The time average of the Poynting vector is:

$$< S >_t = < \mathrm{Re}[E(t)]\mathrm{Re}[H(t)] >_t \hat{i} = \frac{1}{2}\mathrm{Re}[E(t)H^*(t)]\hat{i}$$

The Mixer: Visible and Infrared IV

So that the power falling on a mixer with area A is:

$$< P(t) >= \frac{A}{2}(\frac{\varepsilon}{\mu})^{1/2} | E(t) |^2$$

substituting back in to retrieve the explicit time dependence:

$$E_T(t) = E_L(t) + E_S(t) = E_{0L}e^{-i\omega_L t} + E_{0S}e^{-i\omega_S t}$$

we have:

$$< P(t) >= \frac{A}{2}(\frac{\varepsilon}{\mu})^{1/2}[(E_{0L}e^{-i\omega_L t} + E_{0S}e^{-i\omega_s t})(E_{0L}^* e^{i\omega_L t} + E_{0S}^* e^{i\omega_s t})]$$

$$= \frac{A}{2}(\frac{\varepsilon}{\mu})^{1/2}[| E_{0L} |^2 + | E_{0S}^2 | + E_{0S}E_{0L}^* e^{-i(\omega_S - \omega_L)t} + E_{0S}^* E_{0L} e^{i(\omega_S - \omega_L)t}]$$

The Mixer: Visible and Infrared V

Recall the photo current for photoconductors:

$$I_{ph} = \phi q \eta G$$

photon flux [photons s^-1]; charge of electron; quantum efficiency; photoconductive gain

and the photo-power is:

$$P_{ph} = \phi h\nu = \frac{\phi hc}{\lambda}$$

So that:

$$I(t) = \underbrace{\frac{\eta q G}{h\nu}}_{\text{response [amps/watt]}} P(t)$$

Heterodyne: 1 | 15 | A525 – Lecture 30

The Mixer: Visible and Infrared VI

Substituting back in we get:

$$I(t) = I_L + I_S + 2(I_L I_S)^{1/2}\cos[(\omega_L - \omega_S)t + \phi]$$

D.C. current from Laser ⇔ $$I_L = \frac{\eta q G A}{2h\nu}\left(\frac{E}{\mu}\right)^{1/2} |E_{0L}|^2$$

D.C. current from source ⇔ $$I_S = \frac{\eta q G A}{2h\nu}\left(\frac{E}{\mu}\right)^{1/2} |E_{0S}|^2$$

The relative phase, φ, is given by:

$$\phi = \arctan\left[\frac{\mathrm{Re}[E_{0S}]\mathrm{Im}[E_{0L}] - \mathrm{Im}[E_{0S}]\mathrm{Re}[E_{0L}]}{\mathrm{Re}[E_{0S}]\mathrm{Re}[E_{0L}] + \mathrm{Im}[E_{0S}]\mathrm{Im}[E_{0L}]}\right]$$

Heterodyne: 1 | 16 | A525 – Lecture 30

The Mixer: Visible and Infrared VII

So the photocurrent contains D.C. terms and a component that oscillates at a frequency:

$$\omega_{IF} = | \omega_L - \omega_S |$$

The IF current is the heterodyne signal, and has *a mean square* amplitude of:

$$< I_{IF}^2 >_t = 2 I_L I_S \qquad \text{recall: } < \cos^2 \omega t. > = \frac{1}{2}$$

The signal strength therefore depends of the LO power. An increase in signal power occurs when down converting the signal frequency

– this is good -- some forms of noise can be overcome

Conversion Gain: I

Increase in signal strength during down-conversion is called conversion gain, Γ_c, defined by the IF output power delivered to the next stage divided by the input signal power.

Maximum power transfer occurs when input impedance = output impedance, R ⇒ half power transferred. For a photoconductor mixer, then:

$$\Gamma_C = \frac{\text{deliverable IF signal power}}{\text{input signal power}} = \frac{\frac{1}{2} 2 I_L I_S R}{P_S}$$

$$= \frac{\left(\eta q G \big/ h\nu \right) P_S I_L R}{P_S} = \frac{\eta q G}{h\nu} V_b$$

$$\text{Assuming } I_{detector} = I_L \Rightarrow I_L = \frac{V_b}{R} \qquad (I_L >> I_S)$$

Conversion Gain: II

$$\Gamma_C = \frac{\eta q G}{h\nu} V_b$$

For a photoconductor mixer at 10 μm, with V_b= 5V; η = 0.5, G=0.5 we get Γ_c =10.

- In practice, the conversion gain is limited by the saturation of the mixer by LO power
- High Γ_c is useful for overcoming noise in post-mixer electronics.
- The phase of the output signal is related to the phase of the input signal, phase is preserved $\Rightarrow$ can make interferometers which depend on knowing the phase of signals from widely separated telescopes

Fundamental Limits on Noise: I

Need to distinguish between two types of noise:

(1) Those that are independent of LO current, I_L
(2) Those that depend on I_L

In principle, noise type (1) can be eliminated by raising the LO power: However, there is a practical limit with detector saturation and finite LO power available.

Noise type (2) includes those arising from fundamental limits:

- Noise in the mixer from generation (and recombination for photoconductors) of charge carriers by the LO power
- Noise from the thermal background detected by the system

Fundamental Limits on Noise: II

G-R Noise. Assume we increase the LO power so that:

$I_L = I_{LO} >> I_{Source}$ & $I_{LO} >> I_{Background}$

Recall the LO current is: $I_L = \frac{\eta q G}{h\nu} P_L$

Define the frequency bandwidth over which the mixer operates:

$\Delta f_{IF} = \frac{1}{2t_{\text{int}}}$ where t_{int} is the integration time

From the discussion of photoconductors and photodiodes:

$$< I_{G-R}^2 >= (\frac{aq}{t_{\text{int}}}) < I_{ph} > G = 2aqI_L\Delta f_{IF}G$$
$$= \frac{2aq^2\eta G P_L \Delta f_{IF}}{h\nu}$$

Where we have simplified by letting: a = 2 for photoconductor
a = G =1 for photodiode

Fundamental Limits on Noise: III

The background noise current will be:

$$< I_B^2 >= 2I_L I_B$$
$$= 2\frac{\eta q G}{h\nu} P_L \cdot \frac{\eta q G}{h\nu} P_B \qquad (1)$$

For a background power source at temperature T_B, with emissivity ε, the background power is given by:

$$P_B = \underbrace{\frac{1}{2}}_{\text{one polarization}} \overbrace{B_\nu}^{\frac{2h\nu^3}{c^2(e^{h\nu/kT_B}-1)}} (T_B)\varepsilon \underbrace{A}_{\text{collecting area}} \underbrace{\Omega}_{\text{solid angle accepted}} (\overbrace{2}^{\text{double side-band receiver}} \Delta f_{IF}) \qquad (2)$$

Fundamental Limits on Noise: IV

Now AΩ ≡ the etendue of the system and is typically:

$$A\Omega \approx \lambda^2 = c^2/\nu^2 \qquad (3)$$

Substituting (3) & (2) into (1) we have:

$$<I_B^2> = \frac{4\eta^2 q^2 G^2 \varepsilon P_L \Delta f_{IF}}{h\nu(e^{h\nu/kT_B} - 1)}$$

Signal-to-noise ratio: $<I_B^2>$ & $<I_{G-R}^2>$ are the fundamental noise contributions. We assume all sources of noise not dependent on I_L can be eliminated.

The instantaneous SNR at the output of the IF stage is then:

$$(S/N)_{IF} = \frac{\overset{\text{output power}}{<I_{IF}^2>}}{<I_{G-R}^2> + <I_B^2>}$$

Fundamental Limits on Noise: V

Now: $$<I_{IF}^2> = 2I_L I_S = \frac{2\eta q G P_L}{h\nu} \cdot \frac{\eta q G P_S}{h\nu}$$

So that: $$(S/N)_{IF} = \frac{\dfrac{2\eta^2 q^2 G^2 P_L P_S}{h^2\nu^2}}{\dfrac{2q^2\eta G^2 P_L \Delta f_{IF}}{h\nu}\left[a/G + \dfrac{2\eta\varepsilon}{e^{h\nu/kT_B - 1}}\right]}$$

or: $$(S/N)_{IF} = \frac{\eta P_S}{h\nu \Delta f_{IF}\left[a/G + \dfrac{2\eta\varepsilon}{e^{h\nu/kT_B - 1}}\right]}$$

Since this is independent of P_L, it is the fundamental performance limit for a heterodyne receiver at the output of the IF.

Fundamental Limits on Noise: VI

There are two limiting cases:

(1) Quantum Noise Limit: G-R noise from the LO dominates

(2) Thermal Noise Limit: Noise from the background power dominates

The cross-over point occurs when: $\frac{a}{G} = \frac{2\eta\varepsilon}{e^{h\nu/kT_B} - 1}$

$$\Rightarrow \frac{h\nu}{kT_B} = \ln\underbrace{\left(1 + \frac{2\eta G\varepsilon}{a}\right)}_{\sim 1}$$

each term is ≤ 1

a=1 or 2

$\Rightarrow h\nu \approx kT_B$ so when:

$h\nu >> kT_B$ - quantum noise limited

$h\nu << kT_B$ – thermal noise limited

Noise Equivalent Power: I

For high speed applications (e.g. communications) the SNR at the IF is the figure of merit, and we define the minimum detectable power MDP_{IF} for SNR = 1 at the IF bandwidth:

$$MDP_{IF} = \frac{h\nu\Delta f_{IF}}{\eta}\left[\frac{a}{G} + \frac{2\eta\varepsilon}{e^{h\nu/kT_B} - 1}\right]$$

However, for most applications, a detector stage rectifies & smoothes the IF signal. This stage improves the SNR by $(t_{int})^{(1/2)}$ so that at the output of the smoothing stage:

$$\left(S/N\right)_{out} = \left(S/N\right)_{IF}\left(\frac{\tau_{RC}}{\tau_{IF}}\right)^{1/2}$$

Noise Equivalent Power: II

- The output of the IF stage will be sampled uniformly over $\tau_{IF} \Rightarrow$
- The integrator is usually a single stage RC circuit so that:
- Therefore, in the quantum limit:

$$\tau_{IF} = \frac{1}{2\Delta f_{IF}}$$

$$\tau_{RC} = \frac{1}{4\Delta f_{RC}}$$

$$NEP_H = \frac{h\nu a}{\eta G} \Delta f_{IF} \left[\frac{\tau_{IF}}{\tau_{RC}(1\,Hz)} \right]^{1/2}$$

$$= \frac{h\nu a}{\eta G} \Delta f_{IF} \left[\frac{2\Delta f_{RC}(1\,Hz)}{\Delta f_{IF}} \right]^{1/2}$$

$$= \frac{h\nu a}{\eta G} (2\Delta f_{IF} \Delta f_{RC}(1\,Hz))^{1/2}$$

$$= \frac{h\nu a}{\eta G} (2\Delta f_{IF})^{1/2}$$

NEP refers to unit frequency bandwidth

Noise Equivalent Power: III

Similarly, in the thermal limit:

$$NEP_H = \frac{2h\nu\varepsilon}{e^{h\nu/kT_B} - 1} (2\Delta f_{IF})^{1/2}$$

Notice that we have shown that the effective noise bandwidth of a heterodyne receiver is the geometric average of the predetector & post-detector bandwidth, i.e. $(\Delta f_{IF} \cdot \Delta f_{RC})^{1/2}$

Noise Equivalent Power: IV

Other sources of noise Independent of LO power

- Mixer detects total background power

  even background power outside of the IF bandwidth

  $\Rightarrow$ extra noise
- Johnson noise in the mixer
- LO can contribute noise due to phase or amplitude instability
- Amplifiers in signal chain can contribute noise