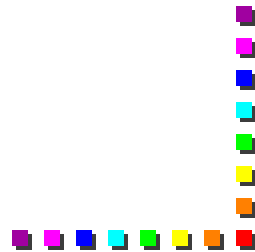


Heterodyne Detection: II

Astronomy 525

Lecture 31



Outline

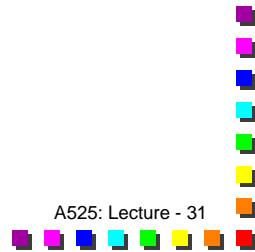
Heterodyne Detection: II

- Performance Attributes: Bandwidth
- Performance Attributes: Throughput
- Noise Temperature
- Quantum Noise and the MASER
- Receiver Noise Temperature
- System Noise Temperature
- Coherent vs. Incoherent Detectors
- Measuring the noise temperature
- Example

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Performance Attributes: Bandwidth I

IF bandwidth, $\Delta f_{IF} = \frac{\Delta \omega_{IF}}{2\pi}$ determines the spectral bandwidth

IF bandwidth is limited by: frequency response of the photomixer or IF amplifiers/circuitry

Photoconductor Mixers: RC time constant is short since LO power forces R down, but recombination times can be long.

For example:

$$\text{Ge:Ga mixer: } \Delta f_{IF}^{\max} \leq 10^8 \text{ Hz}$$

$$100 \mu\text{m} \sim 3 \times 10^{12} \text{ Hz} \Rightarrow \frac{\Delta f_{IF}^{\max}}{f} \sim 3 \times 10^{-5} !$$

(or 10 kms⁻¹)

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Performance Attributes: Bandwidth II

InSb Hot Electron Bolometers: InSb HEB have

$$\Delta f_{IF}^{\max} \sim 10^6 \text{ Hz}$$

$$300 \mu\text{m} : f = 10^{12} \text{ Hz} \Rightarrow \frac{\Delta f_{IF}^{\max}}{f} \sim 10^{-6} ! \quad (0.3 \text{ kms}^{-1})$$

- InSb HEBs are used in “scanning” mode for heterodyne spectrometer - scan LO frequency – loose spectral multiplex advantages
- Recently superconducting thin film HEBs have achieved GHz IF bandwidths, however... more later

Heterodyne: II

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Performance Attributes: Bandwidth II

Photodiode Mixers: very fast, but Δf_{IF} still $\leq 1 - 3 \times 10^9$ Hz

$$300 \mu\text{m} : f = 10^{12} \text{ Hz} \Rightarrow \frac{\Delta f_{IF}^{\text{max}}}{f} \sim 1 \times 10^{-3} \text{ (300 kms}^{-1}\text{)}$$

Marginally O.K., but not good enough for galaxies

$$10 \mu\text{m} : f = 3 \times 10^{13} \text{ Hz} \Rightarrow \frac{\Delta f_{IF}^{\text{max}}}{f} \sim 10^{-4} \text{ (3 GHz IF)}$$

useless for continuous sources \Rightarrow spectral lines only

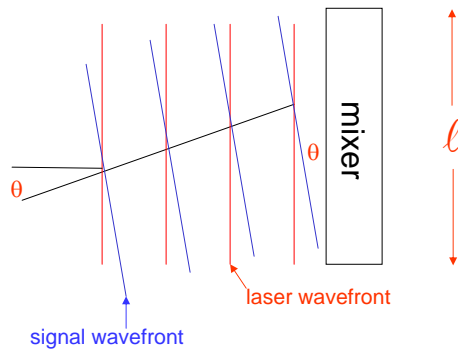
Heterodyne: II

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Performance Attributes: Throughput I

Signal photons can not be concentrated on the mixer in a parallel beam: there is always a range of angles
 \Rightarrow limit to acceptance angle for a heterodyne receiver.



Heterodyne: II

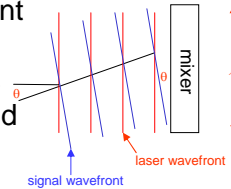
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Performance Attributes: Throughput II

Let's examine the general case where the signal wavefront strikes the mixer at an angle θ w.r.t. the perfectly collimated laser wavefront

There will be a phase shift across the detector between the laser and signal. The resulting IF signal will be fully out of phase when :



$$l \sin(\theta_D) = \frac{\lambda}{2} \cong l \theta_D \leftarrow \theta_{Destructive}$$

Efficient interference (in phase IF signals) will be maintained for \sim half this angle. Recall: $\Omega = \pi \sin^2(\theta)$
 $\Rightarrow \Omega \approx \pi \left(\frac{\lambda}{2l}\right)^2$ but, since $l^2 \sim A \Rightarrow \boxed{\Omega A \sim \lambda^2}$

Performance Attributes: Throughput III

- Etendue is invariant in any aberration free optical system. It can only get larger in the presence of aberrations.
- Since a heterodyne receiver only accepts $\Omega \approx \lambda^2/A$, any aberrations mean losses. Notice that $\Omega \approx \lambda^2/A \Leftrightarrow \theta \approx \lambda/D$
- \Rightarrow (1) Heterodyne systems should operate at the diffraction limit of the telescope: $\theta_{diff} \approx 1.22\lambda/D$
- Obviously, if the receiver accepts smaller angles ($\theta_{rec} < \theta_{diff}$) then energy must be lost. *not a problem at long λ 's*
- Effects that lead to image broadening, e.g. seeing & poor aperture surface quality lead to poor source-receiver coupling.

"aperture efficiency"

Performance Attributes: Throughput IV

(2) The laser power is polarized, so that heterodyne receivers detect only one polarization:

(1) & (2) \Rightarrow single mode detectors - they are sensitive to only one transverse mode of the radiation field.

Heterodyne: II

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Noise Temperature: I

$$NEP_{Het} \rightarrow \underbrace{\frac{h\nu a}{\eta G} (2\Delta f_{IF})^{1/2}}_{\text{quantum limit}}; \underbrace{2 \frac{h\nu \epsilon}{(e^{h\nu/kT_B} - 1)} (2\Delta f_{IF})^{1/2}}_{\text{thermal limit}}$$

- Notice $NEP_{Het} \rightarrow 0$ as $\Delta f_{IF} \rightarrow 0$: true if the LO power can be increased to overcome non-fundamental noise sources
- In practice, this is not so useful, as:

Continuum: $\frac{S}{N} \propto \frac{\Delta f_{IF}}{\Delta f_{IF}^{1/2}} \Rightarrow$ want Δf_{IF} large

Lines: $\frac{S}{N} \approx \text{constant}$ only when $\Delta f_{IF} > \Delta f_{line}$
 $N \propto \Delta f_{IF}^{1/2}$

when $\Delta f_{IF} < \Delta f_{line} \Rightarrow$ continuum situation
 \Rightarrow optimal Δf_{IF} (as before)

Heterodyne: II

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Noise Temperature: II

- How does a heterodyne receiver respond to a continuum source? Introduce concept of noise temperature, T_N :
- T_N : a blackbody at the receiver input at a temperature T_N produces $SNR = 1$...small T_N is good.
- In the thermal limit, we can show that if $\varepsilon = 1$, then: $T_N = T_B$
Substituting $P_S = \varepsilon B_\nu(T_N) A \Omega \Delta f_{IF}$ into:

$$(S/N)_{IF} = \frac{\eta P_S}{h \nu \Delta f_{IF} \left[\frac{a}{G} + \frac{2\eta\varepsilon}{e^{h\nu/kT_B} - 1} \right]}$$

we have: (letting $S/N = 1$): $T_N = \frac{h\nu}{k} \frac{1}{\ln(\varepsilon - 1 + e^{h\nu/kT_B}) - \ln \varepsilon}$

Or, for $\varepsilon \rightarrow 1$, $T_N \rightarrow T_B$

Noise Temperature: III

In the quantum noise limit: $T_N = \frac{h\nu}{k \ln\left(1 + \frac{2G\eta}{a}\right)}$

which in the ideal case ($G = \eta = a = 1$) goes to the quantum limit:

$$T_N = \frac{h\nu}{k}$$

Qualitatively, this result can be seen as a reflection of the wave-particle duality of light (uncertainty principle):

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Noise Temperature: IV

Now: $\Delta E = h\nu\Delta N$ (N photons detected)

and $\Delta t = \frac{\Delta\phi}{\omega} = \frac{\Delta\phi}{2\pi\nu}$ (phase, ϕ)

$$\Rightarrow \Delta E\Delta t = \frac{h\nu\Delta N}{2\pi\nu} \Delta\phi \geq \frac{\hbar}{2}$$

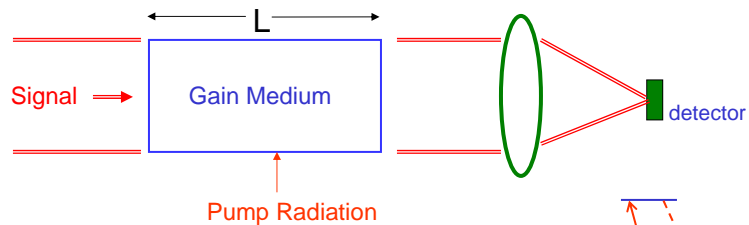
$$\Rightarrow \Delta N\Delta\phi \geq 1/2$$

This can be thought of as a **phase-photon number uncertainty**. The act of coherent detection (small $\Delta\phi$) adds noise (larger ΔN) termed quantum noise.

\Rightarrow In *principle* coherent detection is less sensitive than incoherent detection.



Quantum Noise and the MASER: I



The MASER effects **coherent amplification** by using pump radiation to overpopulate state 2: **population inversion**. The incident radiation then stimulates $2 \rightarrow 1$ emission, and is therefore amplified *in phase*.

c.f. "Limits on Electromagnetic Amplification Due to Complementarity"
 R. Serber & C.H. Townes 1960 in Quantum Electronics, e.d. C.H. Townes
 (NY:Academic Press), p.233



Quantum Noise and the MASER: II

- Of course, any spontaneous emission is amplified as well, and this is what lies at the root of quantum noise. Spontaneous emission is an effect of the uncertainty principle: **it is emission stimulated by fluctuations in the vacuum.**
- Initially, assume the system is in LTE and the pump is off. The gain in an element of length dx is given by: $dP = d\epsilon B_\nu(T) \Delta\nu A\Omega$

where the emissivity element $d\epsilon$, is given by:

$$d\epsilon = \frac{h\nu}{c} (B_{12}n_1 - B_{21}n_2) dx \quad [= da]$$

with B_{jk} being the stimulated absorption and emission coefficients, related to the spontaneous emission coefficient:

$$g_j B_{jk} = g_k B_{kj} = \frac{c^3}{8\pi h \nu^3} g_k A_{kj}$$

Heterodyne: II

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Quantum Noise and the MASER: III

Assuming the $g_i=1$ for this problem we can write:

$$d\epsilon = \frac{c^2}{8\pi\nu^2} A_{21}(n_1 - n_2) dx$$

For real MASERS, only a single diffraction limited spatial mode & polarization is coupled in, so that $A\Omega = \lambda^2$, and we drop the "2" in $B_\nu(T)$:

$$dP = dx \frac{c^2}{8\pi\nu^2} A_{21}(n_1 - n_2) \frac{h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_B} - 1} \Delta\nu \lambda^2$$

Or, since in LTE $\frac{n_1}{n_2} = e^{h\nu/kT_B}$, we have: $dP = \frac{hc^2 \Delta\nu}{8\pi\nu} A_{21} n_2 dx$

Heterodyne: II

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Quantum Noise and the MASER: IV

Now turn on the pump. LTE is destroyed, and dP now contains two terms, the spontaneous term and a term that amplifies the incident power, $P(x)$:

$$dP = \frac{h\nu}{c}(B_{21}n_2 - B_{12}n_1)P(x)dx + \frac{hc^2\Delta\nu}{8\pi\nu} A_{21}n_2 dx$$

$$= \underbrace{\frac{c^2}{8\pi\nu^2} A_{21}(n_2 - n_1)P(x)dx}_{\text{amplified incident power}} + \underbrace{\frac{hc^2\Delta\nu}{8\pi\nu} A_{21}n_2 dx}_{\text{spontaneous emission}}$$

The power exiting the MASER is given by: $\int_0^{P_N} \frac{dP}{\frac{c^2}{8\pi\nu^2} A_{21}(n_2 - n_1)P(x) + \frac{hc^2\Delta\nu A_{21}n_2}{8\pi\nu}} = \int_0^L dx = L$



Quantum Noise and the MASER: V

Or,
$$L = \frac{1}{\frac{c^2}{8\pi\nu^2} A_{21}(n_2 - n_1)} \ln \left[\frac{n_2 - n_1}{n_2 h \nu \Delta \nu} P_N + 1 \right]$$

The output spontaneous emission power is then:

$$P_N = \frac{n_2 h \nu \Delta \nu}{n_2 - n_1} \left(\exp \left[\frac{c^2}{8\pi\nu^2} A_{21}(n_2 - n_1)L \right] - 1 \right)$$

$$= (G - 1) \frac{n_2 h \nu \Delta \nu}{n_2 - n_1}$$

The exponential we just abbreviated by G is called the **power gain of the maser** and depends on A_{21} . If the pump is very strong, most of the atoms will be in state 2, so that:

$$P_N = (G - 1) h \nu \Delta \nu$$



Quantum Noise and the MASER: VI

Now let some signal power, P_s enter the MASER. For this case, then, $S = GP_s$ and the S/N is:

$$S/N = \frac{GP_s}{(G-1)h\nu\Delta\nu} \longrightarrow \frac{P_s}{h\nu\Delta\nu}$$

for very large gains. The appearance of this expression with the denominator equal to the “power” of one photon means:

Coherent detection adds noise equivalent to one photon per unit bandwidth per spatial mode referred to the input of the receiver.

Heterodyne: II

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Quantum Noise and the MASER: VII

Does this matter? Compared with the occupation, \bar{n} , and noise, Δn of photons per unit bandwidth and spatial mode in room temperature radiation:

$$\bar{n} = \frac{1}{e^{h\nu/kT} - 1} \text{ and } \Delta n = (\bar{n}(\bar{n} + 1))^{1/2}$$

At 10 μm : $\Delta n = 0.09 \Rightarrow$ adding noise equivalent to one photon is awful.

At 1 mm: $\Delta n = 21 \Rightarrow$ adding noise equivalent to one photon is not too bad.

But, if $T = 2.7$ K (a space environment):

At 1mm: $\Delta n = 0.07 \Rightarrow$ 1 photon is again quite bad!

Heterodyne: II

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Receiver Noise Temperature: I

Receiver sensitivity is usually expressed in terms of the noise temperature: the temperature of a thermal signal source that produces the same amount of power as the noise.

The power in a blackbody at temperature T_N in a single spatial mode and polarization in the Rayleigh Jeans limit is: $P = kT_N\Delta\nu$ so that:

$$S/N = \frac{P_s}{h\nu\Delta\nu} \Rightarrow 1 = \frac{kT_N\Delta\nu}{h\nu\Delta\nu} \quad \Rightarrow T_N = \frac{h\nu}{k} \quad \text{The quantum noise limit}$$

Similarly, it is convenient to express astronomical source flux densities in terms of the Rayleigh Jeans antennae temperature:

$$P_s = 2kT_s\Delta\nu$$

Where the frequency bandwidth, $\Delta\nu = 2\Delta f_{IF}$ for double side band reception, and we have assumed $A\Omega = \lambda^2$



Receiver Noise Temperature: II

$$T_s = \frac{P_s}{2k\Delta\nu}$$

This relationship holds only for $h\nu \ll kT$, but is useful in general if we term T_s the Rayleigh-Jeans brightness temperature.

The concept of noise temperature is useful in general. We use it to characterize the non-LO dependent sources of noise such as amplifier (following the IF mixer stage) noise.

Amplifier performance is expressed in terms of the Johnson noise of an equivalent "perfect" resistor:

$$\langle I_A^2 \rangle = \frac{4kT_A\Delta f_{IF}}{R_A}$$

↖ amplifier noise temperature
↖ amplifier input resistance



Receiver Noise Temperature: III

The lower limit for this noise figure is **quantum noise**. At 1.5 GHz, the quantum limit is $T_{\text{quantum}} \sim 0.07 \text{ K}$. The best HEMT amplifiers attain: $T_A \sim 2 \text{ K}$. (30 times T_{quantum}).

Assuming that the amplifier noise dominates non-LO dependent sources of noise, we can derive *the minimum LO power required to approach the quantum limit*. We require:

$$\langle I_A^2 \rangle < \langle I_{G-R}^2 \rangle + \langle I_B^2 \rangle$$

0 in quantum limit

$$= \frac{2aq^2\eta GP_L \Delta f_{IF}}{hv}$$

(assumes all resistances equal)

$$P_L > \frac{2kT_A Lh\nu}{q^2 \eta R_A a G^2}$$

loss ($L > 1$) in transferring the mixer output to the amplifier.

Heterodyne: II
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System Noise Temperature: I

The noise temperature of a receiver system contains contributions from: **mixers, amplifiers, thermal backgrounds & other sources**. The total noise is the sum of these contributions referred to the system input:

$$T_N^S = T_M + L_1 T_A + L_1 L_2 T_3 + \dots$$

mixer noise
amplifier noise
noise in 3rd component

loss in passing signal from mixer input to amplifier ($L_1 > 1$) loss in passing signal from amplifier input to 3rd component input

$$L_1 = 1/\Gamma_c = \left(\frac{hv/\eta a}{GV_b} \right) = \frac{\text{input signal}}{\text{delivered IF signal}}$$

photoconductor mixer gain

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System Noise Temperature: II

This formalism is analogous to that for adding noise power in direct detection systems:

$$\langle I_N^2 \rangle = \langle I_{pho}^2 \rangle + \langle I_J^2 \rangle + \langle I_{\gamma}^2 \rangle$$

since

$$\langle I_A^2 \rangle = \frac{4kT_A \Delta f_{IF}}{R_A} \dots$$

The resultant S/N ratio for a coherent receiver is then: (Dicke radiometer equation)

$$\left(\frac{S}{N}\right)_C = K \frac{T_S}{T_N} (\Delta f_{IF} \Delta t)^{1/2}$$

constant related to time response of the output stages -1.
integration time

Coherent vs. Incoherent Detection: I

Keeping in mind the bandwidth and single mode detection restrictions of coherent receivers we may compare coherent w/ incoherent detection:

$$(S/N)_i = \frac{\overbrace{2kT_S \Delta \nu}^{\text{source power } P_s} (\Delta t)^{1/2}}{NEP}$$

Dividing the coherent detection expression by the incoherent expression:

$$\frac{(S/N)_c}{(S/N)_i} = \frac{NEP (\Delta f_{IF})^{1/2}}{2kT_N^S \Delta \nu}$$

Coherent vs. Incoherent Detection: II

As an illustration, take the case where we are using a bolometer in the background limit, and a heterodyne receiver in the thermal limit. Both accept $A\Omega = \lambda^2$, and look at T_B with $\varepsilon = 1$:

Assuming $h\nu \ll kT_B$, then:

$$\frac{(S/N)_c}{(S/N)_i} = \left[\left(\frac{1}{\eta} \right) \left(\frac{\Delta f_{IF}}{\Delta \nu} \right) \left(\frac{h\nu}{kT_B} \right) \right]^{1/2}$$

so that the bolometer wins unless: $\left(\frac{\Delta f_{IF}}{\eta \Delta \nu} \right) \gg 1$

i.e. if the spectral resolution is much bigger than the IF bandwidth, and the incoherent receiver is not spectrally multiplexing. In other words narrow lines -- **more later...**

Heterodyne: II

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Coherent vs. Incoherent Detection: III

On the other hand, if the bolometer is **detector noise limited** and the heterodyne receiver operates in the **quantum limit** we have:

$$\frac{(S/N)_c}{(S/N)_i} = \frac{NEP(\Delta f_{IF})^{1/2}}{2h\nu\Delta\nu}$$

If $R \equiv \nu/\Delta\nu$ is constant, then: $\frac{(S/N)_c}{(S/N)_i} \propto 1/\nu^2$

\Rightarrow there is transition to the realm where coherent receivers win when **frequencies get small**. There also will be spectral multiplex advantage (**in many cases**) for the heterodyne receiver.

But, if you stay background limited, direct detection will always win - for "perfect" receivers.

Heterodyne: II

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Measuring the Noise Temperature

It is fairly straightforward to characterize the noise temperature of a coherent receiver by placing a “hot” & a “cold” load in front of the receiver, blocking the beam. The “Y-factor” is then:

$$Y = \frac{V_{hot}}{V_{cold}}$$

output voltages of the receiver.

If the receiver is linear in power, the output voltages are proportional to the sum of the antenna temperatures of the loads and the noise temperatures of the receiver:

$$Y = \frac{T_{hot} + T_{rec}}{T_{cold} + T_{rec}}$$

$$\Rightarrow T_{rec} = \frac{T_{hot} - YT_{cold}}{Y - 1}$$

Heterodyne: II

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Example: I

Consider a 10 to 11 μm heterodyne receiver viewing a 290 K background with a HgCdTe diode photomixer with the following detector properties:

- $\lambda_{\text{cutoff}} = 15 \mu\text{m}$
- q.e. = 40% for $\lambda < 13 \mu\text{m}$
- size $\sim 100 \mu\text{m} \times 100 \mu\text{m}$
- depletion region width $\sim 1 \mu\text{m}$ at operating bias
- reverse bias impedance $\sim 150 \Omega$
- dielectric constant = 10

And receiver properties:

- LO: CO_2 laser reflected off 10% reflectivity diplexer
- Diplexer transmits 90% of incoming signal
- Photomixer output to 150Ω input impedance amplifier
- Amplifier Gain = 100, BW = 3×10^9 Hz, and
- output noise current: $\langle I_N^2 \rangle^{1/2} = 33.2 \mu\text{A} / \text{Hz}^{1/2}$

Heterodyne: II

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Example: II

The receiver is tested by looking at hot and cold sources

$$500 \text{ K source: } V_H = 1.040 \text{ V}$$

$$300 \text{ K source: } V_C = 1.000 \text{ V}$$

(a) IF bandwidth: Is it limited by amplifier or the RC time constant of the mixer?

$$C = \frac{A \kappa_o \epsilon_o}{w} = 6.95 \times 10^{-13} \text{ Farad}$$

Dielectric constant
Area of diode
Depletion width

$$\Rightarrow RC = 6.95 \times 10^{-13} \times 150 = 1.04 \times 10^{-10} = \tau_{RC}$$

$$\Rightarrow f_{RC} = 1/(2\pi\tau) = 1.5 \times 10^9 \text{ Hz} < 3 \times 10^9 \text{ Hz}$$

\Rightarrow IF bandwidth limited by photomixer response

Heterodyne: II

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Example: III

(b) Does the receiver operate in the thermal or quantum noise limit?

At $10.5 \mu\text{m}$, $h\nu/kT \sim 4.7 > 1 \Rightarrow$ we expect to be in the quantum limit.

More precisely is: $\frac{a}{G} ? \frac{2\eta\epsilon}{\left(e^{\frac{h\nu}{kT_b}} - 1\right)}$

Assume $a = G = 1 \Rightarrow 1 \gg 0.007\epsilon \Rightarrow$ quantum limit

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Example: IV

(c) What is the noise temperature of the amplifier?

We need to convert the output noise figure to an input referred noise:

$$\langle I_A^2 \rangle^{1/2} = \frac{3.2 \times 10^{-5} \text{ A Hz}^{-1/2}}{100} = 3.32 \times 10^{-7} \text{ A Hz}^{-1/2}$$

From: $\langle I_A^2 \rangle^{1/2} = \frac{4kT_A \Delta f_{IR}}{R_A}$

we have: $T_A = \frac{\langle I_A^2 \rangle^{1/2} R_A}{4k \Delta f_{IR}} = \boxed{200 \text{ K}}$

150 Ω (pointing to R_A)
1.50 × 10⁹ Hz (pointing to Δf_{IR})

Example: V

(d) What is the laser power required to overcome the amplifier noise?

To be quantum noise limited, we must have:

$$P_L > \frac{2kT_A L h \nu}{q^2 \eta R_A a G^2}$$

Since the output of the mixer is impedance matched to the input of the amplifier, $L = 1$. For a photodiode, $G = a = 1$, and with $\eta = 40\%$, we have: $P_L > 0.068 \text{ mW}$

Since only 10% of the laser is reflected into the mixer by the diplexer, we need: $P_L > 0.68 \text{ mW}$

Example: VI

(e) If $P_L = 20$ times 0.68 mW (the minimum), what is the conversion gain?

The conversion gain will be: $\Gamma_c = \left(\frac{\eta q}{h\nu} \right) I_{ph} R$

For $P_L = 13.6$ mW, $P_L(\text{reflected}) = 1.36$ mW, so that with $\eta = 0.40$ this produces a current of 2.88×10^{16} e-/sec, or 4.61 mA $\Rightarrow \Gamma_c = 2.34$

Example: VII

(f) What is the expected system noise temperature?

$$T_M = \frac{h\nu}{k \ln\left(1 + \frac{2G\eta}{a}\right)} = \frac{1370}{\ln(1+0.4)} = 4077 \text{ K}$$

$$T_N^S = T_M + L_1 T_A + L_1 L_2 T_3 + \dots$$

$$L_1 = \frac{1}{\Gamma_c} = 0.427 \Rightarrow T_N^S = 4163 \text{ K}$$

Example: VIII

(g) What is the measured system noise temperature?

$$Y = \frac{V_{hot}}{V_{cold}} = \frac{1.040}{1.000} = 1.040$$

$$T_{rec} = \frac{T_{hot} - Y \cdot T_{cold}}{Y - 1} = \frac{500 - 1.040 \cdot 300}{1.040 - 1.000} = 4700 \text{ K}$$

There is excess noise at the level of $4700 - 4163 = 537 \text{ K}$.

(h) If there are 50 strong laser lines in the 10 to 11 μm regime, what is the probability that the receiver can operate at an arbitrary frequency between 10 and 11 μm ?

Bandwidth per line = $\pm \Delta f_{IF} = 3 \text{ GHz} \times 50 \text{ lines} = 1.5 \times 10^{11} \text{ Hz} \sim 5.5\%$ of the 10 to 11 μm band. Therefore, 5.5% probability.

