1. Inertial oscillations. The governing equations are
\[ \frac{Du}{Dt} = fv, \quad \text{(1)} \quad \text{and} \quad \frac{Dv}{Dt} = -fu. \quad \text{(2)} \]
The particle starts with a positive eastward speed \( V_0 \) and, we may assume, with no northward motion. We learned by substitution during lecture that sine or cosine of \( ft \) give solutions for the velocity components. In this case a cosine is suggested for \( u \), since \( u \) is positive at \( t = 0 \). Substitution verifies that (1) and (2) are satisfied by
\[ u = V_0 \cos ft, \quad v = -V_0 \sin ft. \]
Now use
\[ \frac{Dx}{Dt} = u = V_0 \cos ft, \quad \frac{Dy}{Dt} = v = -V_0 \sin ft. \]
Integrate to find \( x(t) \) and \( y(t) \). Choose the constants of integration to be zero, so that the particle orbits are centered at \( x = 0, y = 0 \). This gives
\[ x = \frac{V_0}{f} \sin ft, \quad y = \frac{V_0}{f} \cos ft. \]
Inspection shows that these indeed describe circular motion, with radius \( \frac{V_0}{f} \). Using \( V_0 = 1 \text{ cm s}^{-1} \) and \( f = 0.73 \times 10^{-4} \text{ s}^{-1} \) gives a radius of about 137 m for the oceanographic oscillations.

2. Problem 1.13 in Holton. The thickness of a layer between \( p_1 \) and \( p_2 \) is given by
\[ z_2 - z_1 = R \frac{g}{T} \ln \frac{p_1}{p_2}. \]
Therefore if \( \langle T \rangle \rightarrow \langle T \rangle + \delta T \), the thickness will change by
\[ \delta z = R \frac{\delta T}{g} \ln \frac{p_1}{p_2}. \]
Solving for \( \delta T \) and plugging in numbers gives a layer mean temperature change of 2.96 K corresponding to a 60 m thickness change.

3. Problem 1.14 in Holton. A uniform density atmosphere. To find height, the strategy should be to evaluate \( dz \) from the hydrostatic equation and integrate. Thus
\[ dz = -\frac{dp}{\rho g}. \]
Since \( \rho \) is constant this is easily integrated. Carry the integral from the base of the atmosphere \( z_0 = 0, p_0 \) to a higher level \( z_1, p_1 \). One finds
\[ z_1 = \frac{p_0 - p_1}{\rho g}. \]
Then use the ideal gas equation to evaluate density in terms of temperature and pressure at the base of the atmosphere, \( \rho = p_0 / (RT_0) \). Substitute, and
Thus if \( p_1 = 0 \), as at the top of the atmosphere, the height is \( z_1 = \frac{RT_0}{g} \), which depends only on the surface temperature. For \( T = 273 \) K, the height is about 8 km.

4. Problem 1.15 in Holton. Temperature in the atmosphere of the previous problem. Since density is constant, the ideal gas law implies that temperature is proportional to pressure. Thus

\[
\frac{T}{T_0} = \frac{p}{p_0},
\]

where \( T_0 \) and \( p_0 \) are values at the base. From the preceding problem,

\[
z = \frac{p_0 - p}{p_0} \frac{RT_0}{g} = \left(1 - \frac{p}{p_0}\right) \frac{RT_0}{g}
\]

Solving for \( p/p_0 \) and equating the result to \( T/T_0 \) gives

\[
\frac{p}{p_0} = \frac{T}{T_0} = 1 - \frac{g}{RT_0} z.
\]

This is OK as a solution. Other forms that are clear are

\[
T = T_0 - \frac{g}{R} z \quad \text{or} \quad T = T_0 \left(1 - \frac{z}{H}\right),
\]

where \( H = \frac{RT_0}{g} \) is the scale height based on the surface temperature.

5. Problem 1.16 in Holton. Height as a function of pressure with a uniform lapse rate. Let \( p_0, \ T_0 \) be surface values, and let \( z = 0 \) at the surface. A constant lapse rate \( \gamma \) implies that \( T = T_0 - \gamma z \). Since we need to solve for height, invert the hydrostatic equation to get \( dz \), and then use the equation of state to eliminate density. We find

\[
dz = -\frac{dp}{\rho g} = -\frac{RT}{g} \frac{dp}{p} = -\frac{R}{g} (T_0 - \gamma z) \frac{dp}{p}.
\]

Then divide by the factor with \( z \) in it, to get all the references to \( z \) on the left side and all the references to \( p \) on the right side. Then multiply by \( -\gamma \) to get a perfect differential on the left side. This gives

\[
-\gamma dz = \frac{\gamma R}{g} \frac{dp}{p}
\]

The left side is now in the form \( du/u \), where \( u = T_0 - \gamma z \). Integrate from the base of the atmosphere up to the level \( z_1, \ p_1 \). One finds

\[
\ln \left(\frac{T_0 - \gamma z_1}{T_0}\right) = \frac{\gamma R}{g} \ln \left(\frac{p_1}{p_0}\right)
\]

Taking the anti-log and solving for \( z_1 \) gives

\[
z_1 = \frac{T_0}{\gamma} \left[1 - \left(\frac{p_1}{p_0}\right)^{\gamma R/g}\right].
\]