1. (a) \( C = 2\pi Rv = 3.14 \times 10^7 \text{ m}^2\text{s}^{-1} \)
(b) Half as large -> one half the circulation.
(c) Double the velocity -> twice the circulation.
(d) By convention, counterclockwise is positive, so reversing -> change sign.
(e) Circulation is \( 2\pi \) times angular momentum per unit mass.
(f) \( 5 \times 10^6 \text{ m}^2\text{s}^{-1} \).

2. Since friction and pressure gradients are negligible, the circulation \( C \) is conserved as the loop drifts inward. Thus \( C_i = C_f \), where the \( i, f \) subscripts refer to initial and final configurations. Initially the loop has no relative velocity, and thus
\[
C_i = 2\Omega \sin \phi \pi R_i^2, \quad C_f = 2\Omega \sin \phi \pi R_f^2 + 2\pi R_f v_f,
\]
where \( v_f \) is the final value of the tangential velocity at the eye wall. Equating and solving for the velocity gives
\[
v_f = \frac{\Omega \sin \phi (R_i^2 - R_f^2)}{R_f}.
\]
Use \( \Omega = 0.19 \times 10^{-4} \text{ s}^{-1} \), appropriate for 15 degrees latitude, \( R_i = 500 \text{ km} \), \( R_f = 100 \text{ km} \), and we find \( v_f = 45.6 \text{ m s}^{-1} \).


![Diagram]

from 1 to 2: \( \vec{u} = -u_o \vec{i}, \quad d\vec{l} = (10^6 \text{ m})\vec{i} \)
from 2 to 3: \( \vec{u} \cdot d\vec{l} = 0 \)
from 3 to 4: \( \vec{u} = -(u_o - 20 \text{ m/s}) \vec{i}, \quad d\vec{l} = (10^6 \text{ m})(-\vec{i}) \)
from 4 to 1: \( \vec{u} \cdot d\vec{l} = 0 \)
\[
C = \int \mathbf{u} \cdot d\mathbf{l} = \int_{1}^{2} -u_0 \, dx + \int_{3}^{4} (u_0 - 20 \text{ m/s}) \, dx \\
= \int_{0}^{10^6 \text{ m}} -u_0 \, dx + \int_{0}^{10^6 \text{ m}} (u_0 - 20 \text{ m/s}) \, dx = (-u_0)(10^6 \text{ m}) - (u_0 - 20 \text{ m/s})(10^6 \text{ m}) \\
= -2 \times 10^7 \text{ m}^2/\text{s} \\
\zeta = \frac{C}{A} = \frac{-2 \times 10^7 \text{ m}^2/\text{s}}{10^{12} \text{ m}^2} = -2 \times 10^{-5} \text{ s}^{-1}
\]


First approach: Absolute circulation.
\[
\pi R_i^2 f + 2\pi R_i v_i = \pi R_f^2 f + 2\pi R_f v_f.
\]
Since \(v_i = 0\),
\[
v_f = \frac{\pi f (R_i^2 - R_f^2)}{2\pi R_f} = fR_f \left(\frac{R_i^2}{R_f} - 1\right) = -\frac{3}{8} fR_f = -5.469 \text{ m s}^{-1}.
\]
Second approach: potential vorticity
\[
r_i = 100 \text{ km and } r_f = 200 \text{ km}
\]
\[
\frac{\zeta + f}{h} = \text{constant}
\]
Initially: \(\zeta_i = 0, \ h = h_i\)
Finally: \(\zeta = \zeta_f, \ h = h_f\) → preserve volume of the cylinder
\[
\pi r_i^2 h_i = \pi r_f^2 h_f
\]
\[
\frac{r_i^2}{r_f^2} = \frac{h_f}{h_i} = \frac{1}{4}
\]
\[
\frac{\zeta_i + f}{h_i} = \frac{\zeta_f + f}{h_f} \Rightarrow \frac{h_f f}{h_i} = \zeta_f + f \Rightarrow \zeta_f = \frac{f}{4} - f = -\frac{3f}{4}
\]

\[\zeta_f = -\frac{3}{4}(2\Omega\sin 30^\circ) = -5.469 \times 10^{-5} \text{ s}^{-1}\]

Now we want to determine what the mean tangential velocity at the perimeter is after expansion.

\[V_f = \frac{C_{rf}}{2\pi r_i} \text{ and } C_{rf} = \zeta_f (\pi r_f^2) \Rightarrow V = \frac{1}{2} \zeta_f r_f, \text{ where } C_{rf} \text{ is the final relative circulation, and}\]

\[V = \frac{\left(-5.469 \times 10^{-5} \text{ s}^{-1}\right) \left(2 \times 10^5 \text{ m}\right)}{2} = -5.469 \text{ m/s} \text{ (anticyclonic)}\]