1. Urban areas are often warmer than the surrounding countryside by about 2°C. This is known as the urban heat island effect. If a city is located near a large body of water, this urban heating can modify regional land/sea breezes. Such a modification of the local circulation has been modeled and observed for LA, Houston, Athens, and Chicago.

Estimate the difference in the land/sea breeze wind velocity for Houston and LA. Assume that in the unperturbed case (no urban heating effect) the difference in the land and sea temperatures is 10°C, with the land warmer than the ocean during the day and vice versa at night. LA is located closer to the water than Houston. Use, say, 10 km for LA and 100 km for Houston, and estimate the difference in velocity over a period of 1 hour. If you need any other numbers to estimate the difference in velocity, just assume a reasonable value. And note that space scales are small enough to justify neglecting Coriolis accelerations.

The following table can be used to organize your calculations:

<table>
<thead>
<tr>
<th></th>
<th>HOUSTON</th>
<th>LOS ANGELES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>w/o urban heat</td>
<td>with urban heat</td>
</tr>
<tr>
<td></td>
<td>day</td>
<td>night</td>
</tr>
<tr>
<td>$T_2 - T_1$</td>
<td>10 K</td>
<td>-10 K</td>
</tr>
<tr>
<td>L (m)</td>
<td>$10^5$ m</td>
<td>$10^5$ m</td>
</tr>
<tr>
<td>$\frac{D \langle v \rangle}{Dt}$ (x$10^{-3}$ m/s)</td>
<td>1.7</td>
<td>-1.7</td>
</tr>
<tr>
<td>$\langle v \rangle$ m/s</td>
<td>6.0</td>
<td>-6.0</td>
</tr>
</tbody>
</table>

In which city is the land/sea breeze stronger, and why? How do you think the urban heat effect on land/sea breezes influences pollution levels in LA and Houston?

Solution:

Here’s the table filled in. Your numbers may be slightly different, because there is flexibility in choosing the depth of the circulation.
The land/sea breeze is stronger in LA because other factors are about equal and LA is closer to the ocean.

During the day, when the wind is blowing onto the shore and into the city, the urban heating effect is stronger so the pollutants are trapped in the city. At night, when the wind blows away from the city and transports pollutants out of the city, the velocity is weakened by the urban heating so the ventilation is less effective. Thus, the urban heating effect leads to higher pollution levels within the city.

2. Problem 4.3 in Holton.
Solution:
\[ \eta_i = f_i + \zeta_i = 2\Omega \sin 30^\circ + 5 \times 10^{-5} \text{ s}^{-1} = 12.29 \times 10^{-5} \text{ s}^{-1} \]
\[ \eta_f = \eta_i = 2\Omega \sin 90^\circ + \zeta_f \]
\[ \zeta_f = \eta_i - 2\Omega = -2.29 \times 10^{-5} \text{ s}^{-1} \]

3. Problem 4.4 in Holton.
Solution:

The barotropic vorticity equation is:
\[ \frac{d}{dt}\left(\frac{\zeta + f}{h}\right) = 0 \]

Therefore:
\[ \frac{\zeta_i + f_i}{h_i} = \frac{\zeta_f + f_f}{h_f} \Rightarrow \frac{2\Omega \sin 60^\circ N}{10 \times 10^3 \text{ m}} = \frac{\zeta_f + 2\Omega \sin 45^\circ N}{7.5 \times 10^3 \text{ m}} \]
\[ \zeta_f = \frac{(2\Omega \sin 60^\circ) \left(7.5 \times 10^3 \text{ m}\right)}{10 \times 10^3 \text{ m}} - 2\Omega \sin 45^\circ = -8.40 \times 10^{-6} \text{ s}^{-1} \]
\[ \eta_f = \zeta_f + f_f = -8.4 \times 10^{-6} \text{ s}^{-1} + 2\Omega \sin 45^\circ = 9.47 \times 10^{-5} \text{ s}^{-1} \]
4. Problem 4.5 in Holton.

Solution: We want the average vorticity for two different areas. The average vorticity is the circulation around the area divided by the area. With the tangential velocity given by \(v = A/r\), the circulation around a circle of radius \(r\) is

\[
C = \oint U \cdot dl = 2\pi r v = 2\pi r \frac{A}{r} = 2\pi A.
\]

This is independent of \(r\)! Thus if we evaluate the circulation around the annular area sketched below, we will get zero, because we need to go around the outer circle in the opposite direction from the inner circle. A possible path around the area is indicated below, one that keeps the area on the left all the way around and is therefore an extension of the counterclockwise principle.

To evaluate the average vorticity within a circular area, use

\[
\zeta = \frac{C}{\pi r^2}.
\]

Numerically, for the inner circle this gives

\[
\zeta = \frac{2\pi A}{\pi r^2} = \frac{2A}{r^2} = 0.5 \times 10^{-4} \text{ s}^{-1}.
\]

Thus for this \(1/r\) swirl pattern, the vorticity in any annulus around the origin is zero, but the average vorticity over any circular area that includes the origin is non-zero and is larger as the area shrinks. The conclusion is that the vorticity is concentrated at the origin, \(r = 0\), where the velocity becomes infinite. In a realistic flow, as for example a hurricane, the central core has finite radius and a smooth vorticity distribution.
5. Problem 4.7 in Holton.
Solution:

The circulation equation is

$$\frac{DC}{Dt} = -\oint RT \, d \ln p$$

Along the north and south sides, $dp = 0$ and there will be no contribution. Along the east and west sides, temperature is constant, and the integration gives simply $\ln p$ evaluated between the end points. We find

$$\frac{DC}{Dt} = -R(T_0 + \Delta T) \ln \left( \frac{p_0 + \Delta p}{p_0} \right) - RT_0 \ln \left( \frac{p_0}{p_0 + \Delta p} \right)$$

$$= -R \Delta T \ln \left( \frac{p_0 + \Delta p}{p_0} \right) = -7.16 \text{m}^2\text{s}^{-2}.$$