1. One day, the synoptic system sketched below was observed over the southwestern USA. The sketch shows geopotential height contours in the x-y plane (local Cartesian coordinates) on the 850 hPa pressure level. The innermost contour is 1500 m, a local high, and the contour interval is 50 m. An air parcel with one corner at \((x_0, y_0)\) is indicated by the square.

(a) Draw arrows indicating the directions of the pressure gradient and Coriolis forces acting on the air parcel, as well as the wind velocity at \((x_0, y_0)\).

(b) Will the wind speed be greater, smaller, or about the same at \((x_0, y_1)\)? Explain briefly.

Greater at \((x_0, y_1)\), because contours closer together implies a larger height gradient.

2. Derive an equation for \(\frac{dT}{dz}\) along the path of a rising bubble of atmosphere, assuming that no heat is added during the ascent and that the atmosphere is in hydrostatic equilibrium.

\[
\rho c_p \frac{DT}{Dt} - \frac{dp}{Dt} = 0.
\]

\[
\therefore \rho c_p \frac{dT}{dt} = dp = -\rho g \, dz
\]

\[
\therefore \, c_p \, dT = -\rho g \, dz
\]

\[
\therefore \, \frac{dT}{dz} = -\frac{g}{c_p}
\]
3. By (about) how much would the distance between the 200 and 300 hPa levels in Jupiter’s atmosphere change if the average temperature between them were to increase by 10K? The universal gas constant is \( R_U = 8.314 \text{ J K}^{-1} \text{ mole}^{-1} \) and the molecular weight of Jupiter’s hydrogen and helium atmosphere is \( m = 0.023 \text{ kg mole}^{-1} \). Given on blackboard: on Jupiter \( g = 26 \text{ m s}^{-2} \).

\[
\begin{align*}
z_2 - z_1 &= \frac{R}{g} \int_{p_1}^{p_2} T \, d\ln p = \frac{R \langle T \rangle}{g} \ln \frac{P_1}{P_2} \\
\therefore \delta(z_2 - z_1) &= \frac{R \delta \langle T \rangle}{g} \ln \frac{P_1}{P_2} \\
&= \frac{(8.314 \text{ J mole}^{-1} \text{ K}^{-1})(10 \text{ K})}{(0.023 \text{ kg mole}^{-1})(26 \text{ m s}^{-2})} \ln \frac{300}{200} \\
&= 56.4 \text{ m}
\end{align*}
\]

4. A new road is being constructed. One section of the road has a curve with a radius of curvature of 500 m. By how much must the road be banked so the net force acting on a car traveling along the road at 20 m/s (~45 mph) is perpendicular to the roadway? (Note: we are asking for the angle of tilt of the roadway for banking.)

Outward acceleration is centrifugal, downward is gravity. The perpendicular to the roadbed tilts away from the vertical by the angle whose tangent is (outward/downward) = (centrifugal/gravity). This tilt angle is the same as the roadbed slope.

Numerically, the angle comes out to be about 4.57 degrees.
5. Calculate the u-momentum tendency due to advection when:
\[ \vec{v} = 2y \hat{i} + 6 \hat{j} \text{ (m s}^{-1} \text{)} \] (local Cartesian Coordinates).

Tendency means the time rate of change at a fixed location, in this case \( \partial u / \partial t \). The advection contribution is given by
\[ \frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} \]

These are the terms that appear on the right hand side if one solves the momentum equation for \( \partial u / \partial t \).

In this case we are given that \( u = 2y \) and \( v = 6 \), both in meters per second. Substituting, one finds that
\[ \frac{\partial u}{\partial t} = 0 - 6 \times 2 = -12 \text{ m s}^{-2} \]

6. The vertical component of the momentum equation is reproduced below. The mean atmospheric structure is assumed in hydrostatic balance, and this balance has been subtracted out. Density and pressure have been replaced by
\[ \rho = \rho_0(z) + \rho' \]
\[ p = p_0(z) + \rho' \]
a) Assume that \( \rho' / \rho_0 \sim 10^{-2} \) and that \( \rho' \sim 10 \text{ hPa} \). Show that in a scaling sense, the pressure gradient term and the buoyancy term are of the same order. This can be done all at once, including part (b) below, by making size estimates term by term across the equation.

\[
\begin{align*}
\frac{Dw}{Dt} &= -\frac{1}{\rho_0} \frac{\partial \rho'}{\partial z} - g \frac{\rho'}{\rho_0} + 2\Omega u \cos \phi + \frac{u^2}{a} + \frac{v^2}{a} \\
\frac{U}{L} &= \frac{\Delta p}{\rho_0 H} g \frac{\Delta \rho}{\rho_0} \quad \Delta U \\
\frac{10^{-7}}{10^6} &= \frac{10^{-2}}{10^3} \quad 10^{-7} \cdot 10^{-2} \quad 10^{-3} \cdot 10 \\
\frac{10^2}{10^7} &= \frac{10^2}{10^7} \quad \text{(all MKS)} \\
\end{align*}
\]

b) Show that for typical synoptic scale motions all other terms in the equation can be neglected in a first approximation. Assume a scaling estimate for vertical velocity \( W \sim 10^{-2} \text{ m s}^{-1} \). Adopt reasonable values of your own choice for other quantities that you need.
universal gravitational constant $\text{G} = 6.673 \times 10^{-11}$ N m$^2$ kg$^{-2}$

radius of Earth $a = 6371$ km

rotation rate of Earth $\Omega = 7.292 \times 10^{-5}$ s$^{-1}$

acceleration due to gravity at surface of Earth $g_0 = 9.81$ m s$^{-1}$

mass of Earth $M = 5.988 \times 10^{24}$ kg

universal gas constant $R = 8.314$ J K$^{-1}$ mole$^{-1}$

gas constant for Earth's atmosphere $R = 287$ J kg$^{-1}$ K$^{-1}$

density of atmosphere near ground $\rho = 1.225$ kg m$^{-3}$

\[ \frac{\partial u}{\partial t} + \vec{v} \cdot \nabla u = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \phi - 2\Omega w \cos \phi + \frac{uv \tan \phi}{a} - \frac{uv}{a} + F_{rx} \]

\[ \frac{\partial v}{\partial t} + \vec{v} \cdot \nabla v = -\frac{1}{\rho} \frac{\partial p}{\partial y} - 2\Omega u \cos \phi - \frac{u^2 \tan \phi}{a} - \frac{vw}{a} + F_{ry} \]
\[ \delta v = -\Omega u_0 t^2 \sin \phi \]

\[ \frac{\partial w}{\partial t} + \vec{v} \cdot \nabla w = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g + 2\Omega u \cos \phi + \frac{v^2}{a} + \frac{vw}{a} + F_{rz} \]
\[ \frac{Du}{Dt} = \frac{\partial u}{\partial t} + \vec{v} \cdot \nabla u \]

\[ \vec{v}_g = \kappa \times \frac{1}{\rho f} \nabla p = \kappa \times \frac{1}{f} \nabla \Phi \]
\[ z_2 - z_1 = \frac{R}{g} \int_{\rho_1}^{\rho_2} T \, d \ln p \]

\[ \vec{u} = u\vec{i} + v\vec{j} + w\vec{k} \]
\[ u = a \cos \phi \frac{D\lambda}{Dt}; \quad v = a \frac{D\phi}{Dt}; \quad w = \frac{Dz}{Dt} \]

\[ H = \frac{R\langle T \rangle}{g} \quad M = [\Omega a \cos \phi + u]a \cos \phi \]

\[ \Phi = gz \quad p = \rho RT \quad R = \frac{R_{\text{universal}}}{m} \]

\[ \vec{F}_p = -\frac{1}{\rho} \frac{\partial p}{\partial \eta} = -\frac{1}{\rho} \nabla p = -\nabla \rho \Phi \]
\[ \left| \vec{F}_{\text{cent}} \right| = \frac{u_{\text{abs}}^2}{R} = \Omega^2 R \]

\[ \vec{F}_g = -\frac{GMm}{r^3} \vec{r} \]
\[ \frac{dp}{dz} = -\rho g \]
\[ \rho c_p \frac{DT}{Dt} - \frac{Dp}{Dt} = \rho J \]