THE ASTROPHYSICAL JOURNAL, 630:875–878, 2005 September 10

CONFINEMENT OF SUPERNOVA EXPLOSIONS IN A COLLAPSING CLOUD

M. M. KASLIWAL,[1] R. V. E. LOVELACE, AND J. R. HOUCK
Department of Astronomy, Cornell University, 220 Space Sciences Building, Ithaca, NY 14853;
mmk32@cornell.edu, rvl1@cornell.edu, jrh13@cornell.edu
Received 2004 December 10; accepted 2005 May 16

ABSTRACT

We analyze the confining effect of cloud collapse on an expanding supernova shock front. We solve the differential equation for the forces on the shock front due to ram pressure, supernova energy, and gravity. We find that the expansion of the shock front is slowed and in fact reversed by the collapsing cloud. Including radiative losses and a potential time lag between supernova explosion and cloud collapse shows that the expansion is reversed at smaller distances, compared to the nonradiative case. We also consider the case of multiple supernova explosions at the center of a collapsing cloud. For instance, if we scale our self-similar solution to a single supernova of energy 10^{51} ergs occurring when a cloud of initial density 10^2 H cm^{-3} has collapsed by 50%, we find that the shock front is confined to $\sim$15 pc in $\sim$1 Myr. Our calculations are pertinent to the observed unusually compact nonthermal radio emission in blue compact dwarf galaxies (BCDs). More generally, we demonstrate the potential of a collapsing cloud to confine supernovae, thereby explaining how dwarf galaxies would exist beyond their first generation of star formation.

Subject headings: galaxies: dwarf — galaxies: individual (Henize 2-10, SBS 0335−052) — galaxies: starburst — stars: dwarf novae — supernova remnants

1. INTRODUCTION

Blue compact dwarf galaxies (BCDs) are observed to experience intense star formation in a spatially compact region. The timescale for this star formation episode of mass $\sim 10^8\ M_\odot$ is only a few million years. Massive stars have short lifetimes, and thus supernova (SN) explosions may occur concurrently in regions of cloud collapse. It is quite probable that this occurs in BCDs. In this paper, we study the confining effect of cloud collapse on an expanding supernova shock front.

The forces acting on the shock front include those due to supernova energy, ram pressure, and gravity. When a supernova shock front expands into a stationary cloud (Sedov 1946; Taylor 1950; Bisnovatyi-Kogan & Silich 1995), the accreted mass does not contribute to a change in the momentum flux. In a region of cloud collapse, not only is mass accreted at a faster rate, it also contributes to a change in the momentum flux. This additional ram pressure leads to the confinement of a supernova shock front in a collapsing cloud. The potential of ram pressure to confine stellar winds of a noncentral OB star to generate a steady-flow situation has been shown (Dopita 1981). We demonstrate that ram pressure can also confine expanding supernova shock fronts with pressures as high as $\sim 10^{-9}$ dynes cm^{-2}, compared to stellar wind pressure, which is $\sim 10^{-12}$ dynes cm^{-2} (Dopita et al. 1981). Our self-similar solution describes the time evolution of a central supernova explosion in a collapsing cloud.

We start with a discussion of the gravitational collapse of a cloud and then obtain the equation for the expansion of a SN shock in the cloud. We first neglect radiative losses and then include them. Next, we consider a time lag between when the supernova explodes and when the cloud begins to collapse. Finally, we consider the case of multiple SN explosions occurring at the center of a two-component collapsing cloud. The inner component has higher density and collapses rapidly to produce the central stellar population. A few million years later, the resulting supernova shock fronts collide with the infalling, less dense, outer component of the cloud.

2. MODEL

We first consider a simple model for the free-fall gravitational collapse of a cloud with negligible pressure. The gravitational force on a differential spherical shell with initial radius R_0 and radius $R(t)$ is

$$dM(R)\frac{d^2R}{dt^2} = -\frac{GM(R_0)dM(R)}{R^2}, \tag{1}$$

where G is the universal gravitational constant and $M(R_0) = 4\pi R_0^3\rho_0/3$ is the initial mass enclosed by this shell, where ρ_0 is the initial density of the cloud, which is assumed to be uniform. Solving this equation, assuming the initial velocity is zero, gives

$$\frac{dR}{dt} = -\left[2GM(R_0)\left(\frac{1}{R} - \frac{1}{R_0}\right)\right]^{1/2}. \tag{2}$$

The free-fall time of the cloud is

$$t_{\rm ff} = \int_0^{R_0} \frac{dR}{-dR/dt} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} \approx \left(5.2\times 10^6\ {\rm yr}\right)(n_2)^{-1/2}, \tag{3}$$

where n_2 is density in units of 10^2 H cm^{-3}. The free-fall time is independent of the initial radius. Thus, the collapse is homologous, as shown in Figure 1.

The velocity of the shell hitting the shock front contributes to the ram pressure that slows down the front. We normalize the time and radius as $\tau = t/t_{\rm ff}$ and $\overline{R} = R/R_0$. Hence, we obtain

$$\tau = \frac{2}{\pi}\int_{\overline{R}}^1 dr'\left(\frac{r'}{1-r'}\right)^{1/2}. \tag{4}$$

[1] Current address: Department of Astronomy, California Institute of Technology, MC 105-24, Pasadena, CA 91125; mansi@astro.caltech.edu.

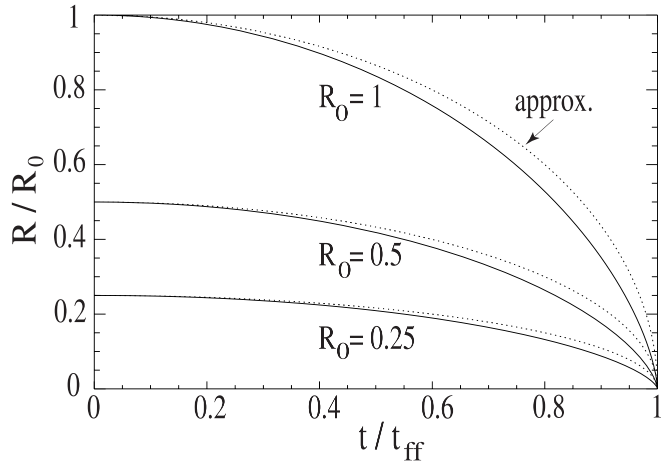

FIG. 1.—Collapse of cloud shells with different initial radii (*solid lines*). The radius is normalized by the initial radius of the outermost cloud shell, and the time is normalized by the free-fall time. The dotted lines indicate the approximation of the trajectory used here. The approximation has a maximum deviation of 5.3% and a mean deviation of 2.6%.

This integral gives a cumbersome expression for $\overline{R}(\tau)$. Therefore, we approximate the dependence as

$$\overline{R} \approx \left(1-\tau^2\right)^{1/2} \equiv f(\tau). \tag{5}$$

Thus, the velocity of the cloud at the shock front radius, $R_{\rm sh}$, is

$$V_{\rm cl}(\tau) = -\frac{\pi R_{\rm sh}[1-f(\tau)]^{1/2}}{2t_{\rm ff}[f(\tau)]^{3/2}}. \tag{6}$$

Accretion of mass just outside the shock front acts to increase in density. Mass conservation gives

$$\Delta M = 4\pi R_0^2 \Delta R_0 \rho_0 = 4\pi R^2 \Delta R \rho(t). \tag{7}$$

Using $R = R_0 f(\tau)$ and $\Delta R = \Delta R_0 f(\tau)$ gives

$$\rho(\tau) = \frac{\rho_0}{[f(\tau)]^3}. \tag{8}$$

There are three forces experienced by the shock front: (1) The energy released by the supernova explosion $E_{\rm SN}$ inside the shell gives an outward-directed pressure $p = (\gamma-1)E_{\rm SN}/[(4/3)\pi R_{\rm sh}^3]$ or force $4\pi R_{\rm sh}^2 p$. We use $\gamma = 5/3$. (2) The ram pressure of the cloud shell hitting the shock front results in an inward-directed force (or pressure). When the directions of motion of the external medium and of the shock front are the same, the ram pressure acts only if the speed of the medium is greater than that of the shock front. This is accounted for by a Heaviside function, H, of the difference between the shock front velocity and cloud velocity [with $H(>0) = 1$ and $H(<0) = 0$]. (3) The gravitational force is, of course, inward.

The ambient pressure of the interstellar medium is $\sim 10^{-10}$ dynes cm^{-2} for a density $\sim$100 H cm^{-3} and temperature $\sim 10^4$ K. For a typical speed $\sim$100 km s^{-1}, the ram pressure is $\sim 10^{-8}$ dynes cm^{-2}. For this reason, we neglect the ambient pressure of the interstellar medium.

The rate at which the SN shock front accretes mass is proportional to the relative velocity between the cloud and the shock front. Thus,

$$\frac{dM_{\rm sh}}{dt} = 4\pi R_{\rm sh}^2 \rho U, \tag{9}$$

where

$$U \equiv \frac{dR_{\rm sh}}{dt} - V_{\rm cl}. \tag{10}$$

The accreted mass initially moves at the cloud velocity. It thus contributes to a net change in the momentum flux. For the case in which the SN explosion and cloud collapse begin at the same time, we have

$$\frac{d}{dt}\left(M_{\rm sh}\frac{dR_{\rm sh}}{dt}\right) = 4\pi R_{\rm sh}^2\left[\frac{E_{\rm SN}}{2\pi R_{\rm sh}^3} + \rho H(U)UV_{\rm cl}\right] - \frac{GM_{\rm sh}^2}{2R_{\rm sh}^2}. \tag{11}$$

We normalize time by the free-fall time and the shock front radius by the Sedov-Taylor radius at the free-fall time (within a factor of 1.24); that is, $\overline{R}_{\rm sh} = R_{\rm sh}/R_{\rm ST}(t_{\rm ff})$, where

$$R_{\rm ST}(t_{\rm ff}) \equiv \left[(\gamma-1)E_{\rm SN}t_{\rm ff}^2/\rho_0\right]^{1/5} \approx (52\ {\rm pc})(E_{51})^{1/5}(n_2)^{-2/5}. \tag{12}$$

We normalize mass by $\overline{M}_{\rm sh} = M_{\rm sh}/[4\pi\rho_0 R_{\rm ST}^3(t_{\rm ff})/3]$. We define a "momentum" variable,

$$P \equiv \overline{M}_{\rm sh}\frac{d\overline{R}_{\rm sh}}{d\tau}. \tag{13}$$

Rewriting equations (9) and (11) in terms of the normalized parameters gives

$$\frac{d\overline{M}_{\rm sh}}{d\tau} = \frac{3\overline{R}_{\rm sh}^2}{f^3}\left[\frac{d\overline{R}_{\rm sh}}{d\tau} + \frac{\pi\overline{R}_{\rm sh}(1-f)^{1/2}}{2f^{3/2}}\right], \tag{14}$$

$$\frac{dP}{d\tau} = \frac{9}{4\pi\overline{R}_{\rm sh}} - \frac{3\pi\overline{R}_{\rm sh}^3(1-f)^{1/2}H\left(\overline{U}\right)\overline{U}}{2f^{9/2}} - \frac{\pi^2\overline{M}_{\rm sh}^2}{16\overline{R}_{\rm sh}^2}, \tag{15}$$

where

$$\overline{U} = \frac{P}{\overline{M}_{\rm sh}} + \frac{\pi\overline{R}_{\rm sh}(1-f)^{1/2}}{2f^{3/2}}. \tag{16}$$

Thus, we have a system of three nonlinear coupled first-order equations. For the initial conditions, we assume that at an early time, $R_{\rm sh}$ and $dR_{\rm sh}/d\tau$ are given by the standard Sedov-Taylor solution for a stationary cloud. We use the initial conditions that at $\tau = 10^{-6}$, $\overline{R}_{\rm sh} = 0.0049$, $\overline{M}_{\rm sh} = 1.2\times10^{-7}$, and $P = 0.00024$. Figure 2 shows $\overline{R}_{\rm sh}(\tau)$ and the corresponding Sedov-Taylor solution. The infalling cloud at first slows the shock expansion and later reverses its motion.

3. INFLUENCE OF RADIATIVE LOSSES

In the above analysis we assumed that the energy within the SN shock was constant, but this is valid only for relatively short times. We next take into account the radiative losses using the pressure-driven snowplow model of Cioffi et al. (1988). We find

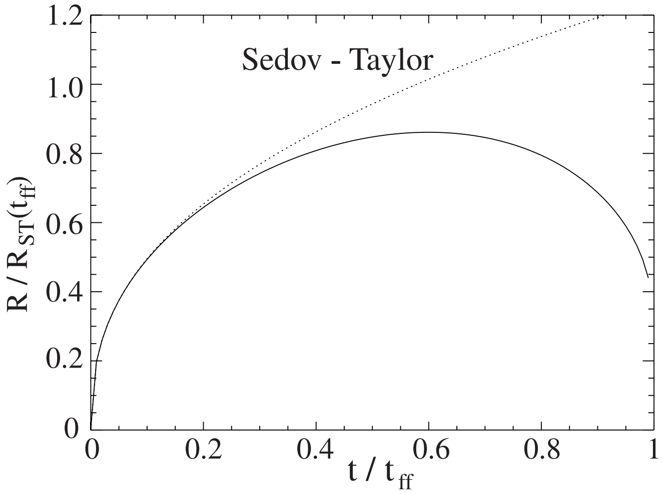

FIG. 2.—Shock wave expansion $R_{\rm sh}(\tau)$ in a collapsing cloud and the standard Sedov-Taylor expansion in a stationary medium. The time τ is normalized by the free-fall time, and the radius is normalized by the Sedov-Taylor radius at the free-fall time. The shock expansion is slowed down and subsequently turned around by the collapsing cloud.

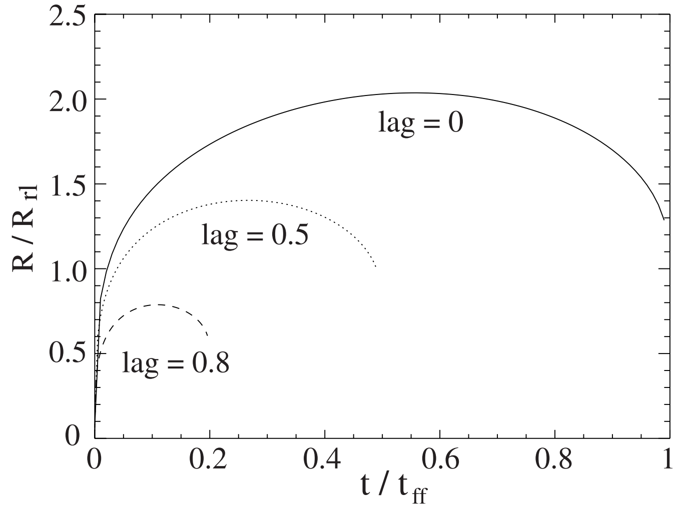

FIG. 3.—Shock front radii including influence of the radiative losses. We also show the shock front radius for a time lag of 50% and 80% of the free-fall time between supernova explosion and cloud collapse. Time zero is taken as the instant the supernova exploded.

that the supernova is confined to a much smaller region in a shorter time. We use the mean pressure $\langle p_{\rm rl}\rangle$ including radiative losses derived by Cioffi et al. (1988), which can be written as

$$\langle p_{\rm rl}\rangle = \frac{\alpha E_{\rm SN}^{5/3}}{R_{\rm sh}^5 t^{4/9}\rho^{10/9}\eta_M^{4/7}}, \qquad (17)$$

where the magnitude of α is 2.3×10^{-17} and η_M is metallicity in units of solar metallicity. Thus, $\langle p_{\rm rl}\rangle$ replaces our earlier pressure $p = E_{\rm SN}/(2\pi R_{\rm sh}^3)$. Next, we assume that the metallicity is solar metallicity and use equation (8) for density as a function of time. The differential equation including the radiative losses is

$$\frac{d}{dt}\left(M_{\rm sh}\frac{dR_{\rm sh}}{dt}\right) = 4\pi R_{\rm sh}^2[\langle p_{\rm rl}\rangle - \rho H(U)UV_{\rm cl}] - \frac{GM_{\rm sh}^2}{2R_{\rm sh}^2}. \qquad (18)$$

We continue to normalize time by the free-fall time. However, we now normalize the radius by

$$R_{\rm rl} = \left(\frac{3\alpha E_{\rm SN}^{5/3} t_{\rm ff}^{14/9}}{\rho_0^{19/9}}\right)^{1/7} \approx (11.8\ {\rm pc})(E_{51})^{5/21}(n_2)^{-26/63}. \qquad (19)$$

We again obtain a self-similar differential equation and solve it using the initial condition that the shock front follows the Sedov solution at $\tau = 10^{-6}$. We use this initial condition because the transition from the Sedov solution to the pressure-driven snowplow radiative shock front occurs at (Cioffi et al. 1988)

$$t_{\rm PDS} \approx \left(1.33\times10^4\ {\rm yr}\right)\frac{E_{51}^{3/14}}{n_0^{4/7}}. \qquad (20)$$

For $E = 10^{51}$ ergs and $n_0 = 10^2$ H cm^{-3}, $t_{\rm PDS} \approx 1700$ yr $\sim t_{\rm ff}/3000$.

The SN explosion may occur at a time Δt after the cloud collapse has begun. The velocity and density of the shells hitting the shock front is then larger. We take the zero of time t or $\tau = t/t_{\rm ff}$ to be the time when the supernova explodes so that only equation (5) needs to be modified and replaced by $f(\tau) = \overline{R} = [1-(\tau+\Delta\tau)^2]^{1/2}$, where $\Delta\tau = \Delta t/t_{\rm ff}$ is termed the "lag."

Figure 3 shows sample results for different lags, which shows that the shock expansion is more strongly decelerated for longer lags. For no time lag, the shock front is confined to $\overline{R}_{\rm sh} = 1.99$ and $\tau = 0.51$. In comparison, for 50% time lag, the shock front is confined to $\overline{R}_{\rm sh} = 1.33$ and $\tau = 0.23$, and for 80% time lag, the shock front is confined to $\overline{R}_{\rm sh} = 0.75$ and $\tau = 0.09$.

4. MULTIPLE SUPERNOVA EXPLOSIONS

We now consider a more complete physical picture. Suppose that there is a localized clump of density 10^4 H cm^{-3} in a cloud of density 10^2 H cm^{-3} and that the ambient outer cloud and the inner clump begin to free-fall. The clump collapses in $\sim$0.5 Myr and forms supernovae in another 2–3.5 Myr (lifetime of a 30–24 $M_\odot$ star). The free-fall time of the cloud is $\sim$5 Myr, and thus the time lag between when the supernova explodes and when the cloud begins to collapse ranges from $0.5t_{\rm ff}$ to $0.8t_{\rm ff}$.

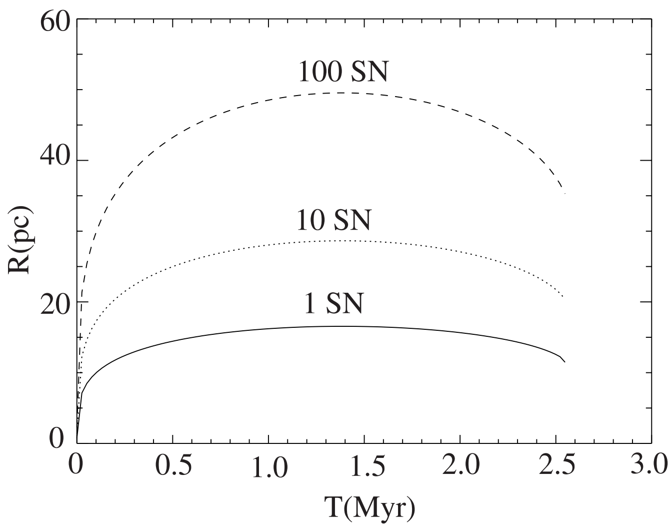

FIG. 4.—Evolution of the shock front radius for single and multiple SN explosions.

When the localized clump collapses, the initial mass function is such that many low-mass (lm) stars form at the same time as a few high-mass stars. The distribution is $dN/dM \propto M^{-2.35}$ (Salpeter 1955). If we take the high-mass (hm) threshold at $>20\ M_\odot$, then $M_{\rm lm} \sim 200 N_{\rm hm}\ M_\odot$. The mass of the low-mass stars contributes to the gravitational force in addition to the forces described in equation (18). The right-hand side of this equation becomes

$$4\pi R_{\rm sh}^2[\langle p_{\rm rl}\rangle - \rho H(U)UV_{\rm cl}] - \frac{GM_{\rm sh}^2}{2R_{\rm sh}^2} - \frac{GM_{\rm sh}M_{\rm lm}}{R_{\rm sh}^2}. \quad (21)$$

We find, however, that the gravitational force contribution of the low-mass stars does not significantly alter the trajectory of the SN shock front. Figure 4 shows the evolution of the shock front radius for single and multiple SN explosions.

5. DISCUSSION

Regarding the stability of the shock front, notice that the Rayleigh-Taylor instability occurs in a stratified region where the effective gravity points in the direction of the less dense medium. In the case of an expanding supernova shock front, the effective gravity in the frame of the shock front is the shock front acceleration. Our results for the trajectory are all concave down, implying that effective gravity points radially outward and thus away from the relatively rarer medium. Hence, the shock front is Rayleigh-Taylor stable.

We conclude that ram pressure from a collapsing cloud reduces the shock front velocity of one radiative supernova explosion (occurring in a localized region when the ambient cloud has collapsed by 50%) to zero at a normalized radius of 1.33 and a normalized time of 0.23. These fractions scale by the energy of the supernova and the density of the interstellar medium.

Observations of nonthermal radio emission in galaxies suggest whether or not the supernovae are confined. For example, a 22 GHz map of M82 extending over a few hundred parsecs is estimated to have 40 unconfined supernovae (Golla et al. 1996). An 18 cm nonthermal radio emission map of VII Zw 19 extending over 310 pc is estimated to have 2500 unconfined supernovae remnants (Beck et al. 2004). Two examples of apparently confined emission, SBS 0335 and Henize 2-10, are described below.

In a model of the nonthermal radio emission from SBS 0335, the emission is confined to 17 pc (Hunt et al. 2004) within a 520 pc region (Thuan et al. 1997) where star formation occurs in six super star clusters with ages $\leq$25 Myr. If a supernova explosion of energy 10^{51} ergs occurs after a cloud of initial density 10^2 H cm^{-3} has collapsed by 50%, it is confined to $\sim$15 pc in $\sim$1 Myr. If this were a supernova exploding in a stationary cloud, it would expand beyond $\sim$20 pc in as little as 0.4 Myr. This would destroy the cloud, and the case of a young supernova remnant in a stationary cloud is considerably less probable than a supernova confined in a collapsing cloud.

Very Large Array imaging of another BCD, Henize 2-10, indicates a $<$8 pc region of 1 mJy radio sources in the central 5$''$ starburst region. Henize 2-10 has H II regions of sizes between 3 and 8 pc and densities between 1500 and 5000 H cm^{-3} (Kobulnicky & Johnson 1999). If we hypothesize that a cloud of an average initial density of 3000 H cm^{-3} has collapsed by 50% when 10 supernovae explode, we find that the supernovae are confined to $\sim$7 pc in $\sim$0.2 Myr. Thus, cloud collapse successfully confines supernova explosions and can account for observed compact nonthermal radio emission. This simple model can help understand continuous or second- and third-generation star formation, since it suggests why the cloud is not devastated by first-generation supernovae.

We thank David Chernoff, Rui Zhen Tan, and Setu Mohta for their valuable comments. We also thank the referee for helpful suggestions. Support for this work was provided by NASA through contract 1257184 issued by Jet Propulsion Laboratory/Caltech. The work of R. V. E. L. was supported in part by Civilian Research and Development Foundation grant KP2-2555-AL-03.

REFERENCES

Beck, S. C., Garrington, S. T., Turner, J. L., & Van Dyk, S. D. 2004, AJ, 128, 1552
Bisnovatyi-Kogan, G. S., & Silich, S. A. 1995, Rev. Mod. Phys., 67, 661
Cioffi, D. F., McKee, C. F., & Bertschinger, E. 1988, ApJ, 334, 252
Dopita, M. A. 1981, ApJ, 246, 65
Dopita, M. A., Ford, V. L., McGregor, P. J., Mathewson, D. S., & Wilson, I. R. 1981, ApJ, 250, 103
Golla, G., Allen, M. L., & Kronberg, P. P. 1996, ApJ, 473, 244
Hunt, L. K., Dyer, K. K., Thuan, T. X., & Ulvestad, J. S. 2004, ApJ, 606, 853
Kobulnicky, H. A., & Johnson, K. E. 1999, BAAS, 31, 1382
Salpeter, E. E. 1955, ApJ, 121, 161
Sedov, L. I. 1946, Dokl. Akad. Nauk SSSR, 42, 17
Taylor, G. I. 1950, Proc. R. Soc. London A, 201, 159
Thuan, T. X., Izotov, Y. I., & Lipovetsky, V. A. 1997, ApJ, 477, 661