Fatal Attraction: Modeling Binary Neutron Stars

Julia S. Deneva ’03, Vassar College
(iudeneva@vassar.edu)
Advisor: James Lombardi, Vassar College

In binary neutron star systems gravity distorts not only the stars, but space itself, leading to gravitational waves. Such waves have already been indirectly detected in the Hulse-Taylor binary, and direct detection is expected in the near future by laser interferometers like LIGO. In order to separate a faint gravity wave signal from local noise observers will need signal templates to which to compare real data. We have performed numerical simulations of neutron binaries by modeling the stars as polytropes and letting the system evolve through a sequence of quasi-equilibrium states. We observed the behavior of different types of systems under different conditions, and tested the accuracy of the code utilizing the Hartle-Thorne theorem. Our ultimate goal is to achieve a good accuracy for both the Newtonian and post-Newtonian versions of the code, so that they can be used to produce gravitational wave signal templates.

I. Introduction and Motivation.

Space can be illustrated by a rubber sheet distorted by mass. A single massive object (like a star) would make an indentation; two stars going around each other would cause the sheet to oscillate in a wave pattern. Merging binary neutron stars are a promising source of gravitational waves and several facilities are being built in the hope of detecting and studying these oscillations of space caused by the extremely dense stars spiraling in towards each other and eventually colliding. As the construction of the Laser Interferometer Gravitational Wave Observatory (LIGO) and other similar facilities progresses, so do theoretical studies of gravitational wave properties. The present work is part of an ongoing project aimed at producing gravitational wave signal templates to be used in LIGO data analysis. We use the same technique as Lombardi, Rasio and Shapiro [3], but the binary neutron star models are constructed numerically rather than semi-analytically. Our goal is to test the Newtonian and post-Newtonian (PN) versions of the code used for the simulations.

The smoothed particle hydrodynamics code (SPH) developed by Rasio and collaborators was used to simulate corotating neutron star binaries in both Newtonian and PN terms on the Cray Origin 2000 parallel supercomputer at NCSA. For an overview of SPH, see Monaghan [4]; for overview of PN SPH calculations, see Faber and Rasio [2]. In Newtonian mechanics, if the orbital separation is greater than the innermost stable circular orbit (ISCO), a binary can exist in equilibrium: the total energy $E = E_{internal} + E_{kinetic} + E_{potential}$ is conserved, and eventual decay is due only to tidal effects. If relativistic effects are taken into account, however, equilibrium cannot be maintained because the generation of gravitational waves causes the system to lose energy. For binaries with large initial orbital separation the decay time due to gravitational wave emissions is much longer than the orbital period, and the stars’ inspiralling can be represented by a sequence of quasi-equilibrium states. The same construct is used for PN simulations since we can again approximate that the orbital decay timescale is much larger than the dynamical timescale.

II. Procedure.

The stars are modeled as polytropes with an adiabatic index $\Gamma=3$. In the equal mass case they are identical, and for a different mass ratio the smaller star is constructed by scaling down the mass of the particles of the larger one. The force experienced by particles includes pressure gradient forces exerted on every given particle by a number of nearest neighbors. In addition, gravitational forces are calculated by an FFT convolution method, and particles which leave the gravity grid as the simulation progresses are treated in a monopole approximation. In this work, the gravity grid spans 8 units of length in each dimension, in radii of the larger star. Resolution can be varied through the number of grid cells, and we tested the code on grid configurations of 64^3, 128^3 and 256^3. Each simulation begins with a relaxation stage, which allows the particles to interact with each other so that the stars can settle into a stable, tidally bulged configuration. A standard relaxation time is 10, in units normalized so that $G = M = R = 1$, where G is the gravitational constant, and M and R are the mass and radius of the larger star, respectively. The system then goes through a sequence of quasi-equilibrium states with the separation slowly decreasing. We choose the orbital frequency necessary for a circular orbit. Since the stars are constrained to be at a certain separation, dynamical instabilities cannot develop and we can investigate even unstable equilibrium states. We force the binary to decay down to, or a little beyond, the separation equal to the ISCO, corresponding to the state in which the system has lowest energy. After this point the system becomes dynamically unstable and the stars merge even if radiation reaction is not included in the calculations. For this reason it was chosen as the end point of our accuracy calculations. We wrote a testing program which uses the equation $\mathrm{d}E = \Omega \mathrm{d}J$ to check whether total energy is conserved, where $\mathrm{d}E$ and $\mathrm{d}J$ are the changes in total energy and angular momentum between consecutive quasi-equilibrium states. It reads in output of the SPH code that contains the values of these quantities for each timestep and returns an error $e = (E_{\mathrm{ISCO}} - E_{\mathrm{initial}})/(\int \Omega \mathrm{d}J) - 1$, where E_{initial} is the energy at the end of the relaxation stage.

III. Results

Figure 1 shows $\mathrm{d}E/\mathrm{d}J$ vs. Ω, with the straight line corresponding to an ideal relationship and the curve illustrating results of a Newtonian simulation. The spike in the leftmost part of the graph corresponds to the end of the relaxation phase, which is ignored in our accuracy calculations. The deviations beginning at approximately $\Omega = 0.26$ correspond to the system approaching instability at the ISCO, where both $\mathrm{d}E$ and $\mathrm{d}J$ approach zero. Figure 3 shows $\int \Omega \mathrm{d}J$ vs. $\int \mathrm{d}E$ for run 5 from Table 1. The dashed line corresponds to an ideal result, and the solid line represents our numerical calculations. The difference between the slopes of the two lines indicates the error of the run.

Table 1 summarizes results from a series of Newtonian simulations. We notice that increasing the total run time does not improve the accuracy significantly, indicating that we are indeed scanning slowly enough for near equilibrium to be achieved at each timestep. Increasing the number of particles or gravity grid size improves accuracy, as expected. The least error is produced by simulations running on grids of size 128 and 256, with the error decreasing as we increase the number of particles. One exception occurs when we increase the gravity grid size from 128 to 256 (runs 5 & 6 and 11 & 12): for both mass ratios used this actually turns out to slightly increase the error.

Movies made of PN runs proved to be especially useful since they revealed anomalies not noticeable in the raw data. In movies with velocity arrows overlaid on the density plot of the stars we noticed that particles had big velocities during the relaxation stage, which should not be the case. The problem turned out to be caused by a bug in the SPH code—the same variable name was used for two different quantities.

IV. Future Work.

The relationship $\mathrm{d}E = \Omega \mathrm{d}J$ is not being satisfied for the PN version of the code, since the total energy until recently has been calculated using only Newtonian terms. We are in the process of adding PN corrections using the derivation of Blanchet, Damour and Schäfer [1] in the expression for total energy. In preliminary runs we found the error to be on the order of 1. It is not yet clear whether this discrepancy is due to a problem with the code, or the means by which the PN energy is calculated. We hope ultimately to perform a similar study on the PN version of the SPH code.

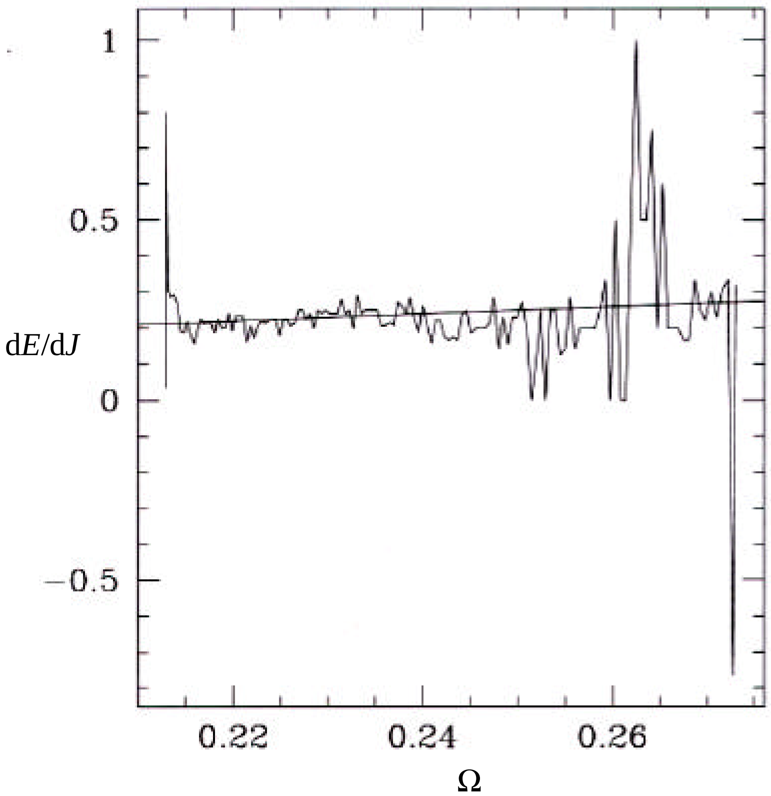

Figure 1—Plot of dE/dJ vs. Ω, for a Newtonian run, where dE and dJ are the changes in total energy and angular momentum between consecutive quasi-equilibrium states. The straight line corresponds to the ideal relationship dE=ΩdJ. Units on both axes are normalized so that G=M=R=1.

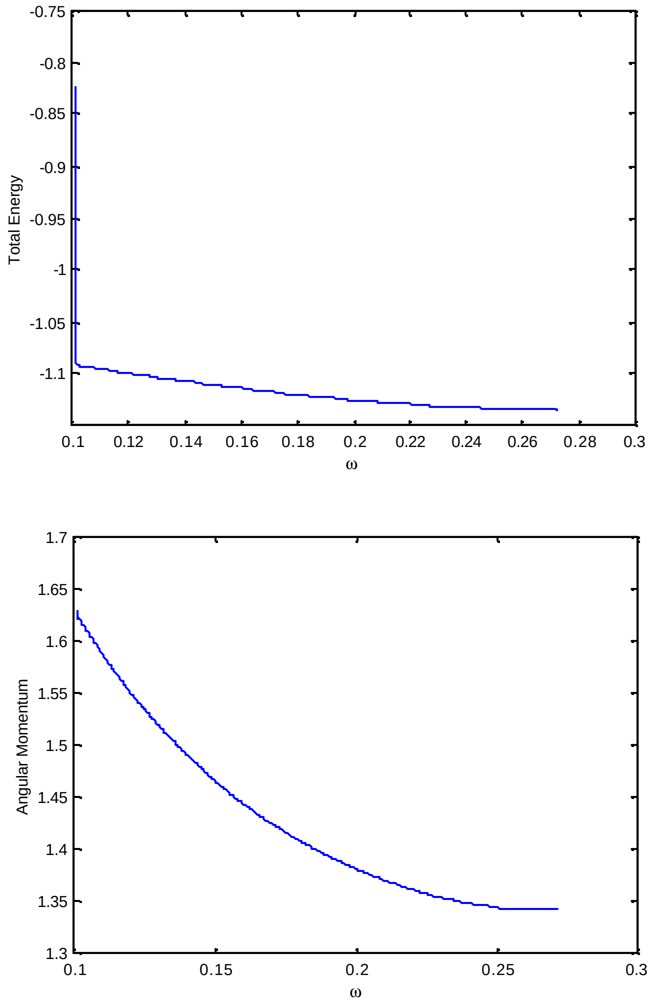

Figure 2—Plots of E_{total} vs. Ω and J vs. Ω for a Newtonian run. The minima of both graphs occur at the same location and correspond to the ISCO.

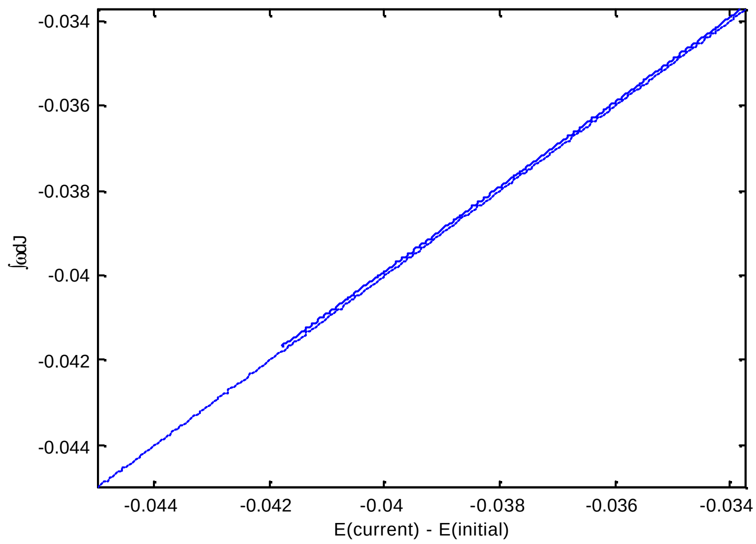

Figure 3—Plot of $\int \Omega dJ$ vs. difference in total energy between the current and the initial quasi-equilibrium states. The dashed line corresponds to an ideal relationship, and the solid line corresponds to numerical results.

Table 1

Run #	Total # of particles	# of grav. grid cells per dimension	Initial separation (in radii of larger star)	Final separation (in radii of larger star)	T_{final}	Mass ratio	Error (e)
1	2000	256	5.7	3.0	200	0.9	-0.02261
2	16000	64	3.5	2.8	200	0.9	-0.02974
3	16000	64	5.7	3.0	200	0.9	-0.01992
4	16000	64	5.7	3.0	800	0.9	-0.01772
5	16000	128	5.7	3.0	200	0.9	0.00366
6	16000	256	5.7	3.0	200	0.9	0.00789
7	2000	256	5.7	3.0	200	1.0	0.01366
8	16000	64	3.7	3.0	200	1.0	-0.04141
9	16000	64	5.7	3.0	200	1.0	-0.01875
10	16000	64	5.7	3.0	800	1.0	0.01878
11	16000	128	5.7	3.0	200	1.0	0.00375
12	16000	256	5.7	3.0	200	1.0	0.00844

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