Determining the Orbital Parameters of Binary Pulsars: Elliptical Systems

Julia S. Deneva, Vassar College ‘03
Adviser: Paulo C. Freire, Arecibo Observatory

Pulsars are very small and dense objects which form when a massive stars explodes as a supernova. They rotate very fast (rotational periods range from a few milliseconds to a few seconds) and emit continuously in a collimated beam which is offset from the rotational axis, and we detect a radio pulse when this beam sweeps past the Earth. Pulsars are generally hard to detect because of scintillation in the atmosphere, and accurate determination of a binary’s orbital parameters is needed in order for the detection to be beneficial for future research. In this work we present the implementation of a novel method for determining the Keplerian orbital parameters of binary pulsars which uses orbital accelerations. Unlike conventional methods for fitting pulsar orbits, the new technique works for very sparse data sets and is independent of the times of individual observations, which makes it more efficient and especially useful in reducing data from observations of weak pulsar binaries where positive detections are few and far apart in time.

Introduction

Studies of binary pulsars has provided verification for numerous important theoretical predictions, among which the Chandrasekhar limit and the existence of gravitational waves. In order for a pulsar detection to be beneficial for further research, the orbital parameters of the system need to be known as a first step. The conventional method for finding orbital parameters involves fitting a Keplerian model to a series of period measurements. However, the accuracy of this method depends on two things which in some cases are not easy to obtain—several period measurements within a single orbit, and a good initial guess for the orbital parameters.

In this work we present the implementation of a new method for determining the Keplerian orbital parameters of binary pulsars developed by Freire at al. (2001) [1]. This method works even for very sparse data sets and has the advantage of being independent of the epochs of individual observations, which makes it especially useful in determining the orbital parameters of weak pulsars where positive detections may be few and far apart in time. For well-sampled pulsars it complements the conventional method by providing a good initial estimate for the orbital parameters.

Theory and Implementation

The time-independent technique is based on two equations expressing rotational period *P* and acceleration along the line of sight *A* as functions of true anomaly, *f*:

$$P(f) = P_0 + \frac{2p P_0 a_P}{P_B\sqrt{1-e^2}}\left(\cos\left(w + f\right) + e\cos w\right) \qquad (1)$$

$$A(f) = -\left(\frac{2p}{P_B}\right)^2 \frac{a_P}{1-e^2}\left(1 + e\cos f\right)^2 \sin\left(w + f\right) \qquad (2)$$

Here P_0 is the intrinsic rotational period of the pulsar, e is the eccentricity, a_p is the projected semi-major axis of the orbit onto a plane which contains the direction towards the Earth and the line of nodes where the orbital plane intersects the plane of the sky, P_B is the orbital period, and ? is the longitude of periastron. If we invert (1) and substitute *f* in (2), we get *A(P)*, a curve which does not depend on time and therefore it is not necessary to solve Kepler’s equation to obtain a fit. If we want to find the last time of passage through periastron, however, we need to solve Kepler’s equation, and that proves to be trivial after we have already found the orbital parameters [2].

In order to get A(P) into a more tractable form, we introduce two new quantities, A_1 and P_1, and make the following substitutions:

$$P_1 = \frac{2pP_0a_p}{P_{BC}} \qquad A_1 = a_p\left(\frac{2p}{P_B}\right)^2$$

For the circular case we can set e = ? = 0, which greatly simplifies the A(P) function—the A vs. P plot is an ellipse centered on (P_0, 0) with P_1 equal to half its horizontal axis, and A_1 equal to half its vertical axis. The implementation for the circular case exploits the properties of this ellipse and a detailed description can be found in [1]. In the elliptical case the A vs. P plot is generally asymmetric, except when ? = np, where n is an integer, and its shape changes a lot depending on ?, so it is not possible to exploit any geometrical properties of the plot. Our code takes as input the periods, accelerations, and times of arrival (TOAs) for individual observations (TOAs are only used when calculating T_0); these are available from pulsar searches. Then we use a custom-written version of the Levenberg-Marquardt least-squares fitting algorithm which picks out the values for the orbital parameters that give the smallest total difference between the fit and the dataset, scaled with the errors of the measurements. The function expressing this difference is:

$$c^2 = \sum_{i=1}^{Ndata} \frac{\left(A_{i,\,data} - A_{i,\,fit}\right)^2}{s_A^{\,2}}$$

In the case of an elliptical system, the $?^2$ function the algorithm must minimize has a lot of local minima, and that is why our solution employs both grid search and numerical derivatives techniques. We either know or can estimate the range for each of the orbital parameters. If x is any one of the parameters we wish to find, we scan its range, evaluating $?^2$ at regular intervals between the minimum and the maximum value that x can take. At the end of the first pass we have an x which minimizes chi-squared; we zoom in on a smaller interval centered on that x and repeat the procedure. What we are aiming for at this stage is a rough estimate of where the minimum of chi-squared is, so this only needs to be done a couple of times and the number of steps taken in each interval can be as low as 5 or 6. Since we have five parameters we wish to find, this is performed in five dimensions. In the next stage we use partial derivatives of chi-squared with respect to the orbital parameters to refine the estimate for the minimum; the set of parameters giving the smallest value of chi-squared also gives the best fit.

Results and Future Work

We tested our code for systems with eccentricities between zero and 0.9, and with randomly generated data sets consisting of 6 to 100 poins. The fits obtained match the A vs. P distributions of the data sets. In the case of extremely sparse data sets there are inaccuracies in the estimate for the orbital parameters due to the absence of “observations” taken near periastron. For data sets of 20 or more points the fitted parameters are very close to the parameters used to generate the data sets, with the values for P_0 being an exact fit within 5 or 6 decimal places and the values for the rest of the parameters within less than one percent to a few percent of their true values (Fig. 1). In order to estimate the error bars of the fitted parameters, we perform a Monte Carlo simulation. We generate 500 data sets, each with the same number of points as the original data set. The generated random accelerations in these sets are Gaussian-distributed and centered on the points in the original data set. Fig. 2 shows the results of a Monte Carlo simulation for a system with eccentricity of 0.6; the values of the parameters used to generate the original data set are within the error bars derived from the simulation.

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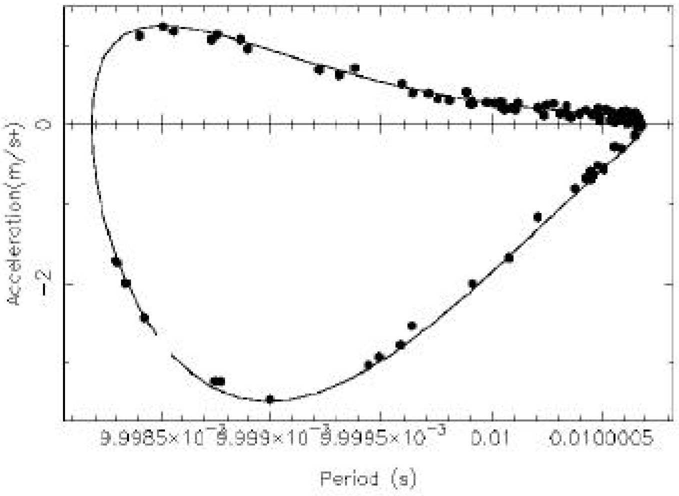

Fig. 1.
Generated data set (dots) parameters:
$P_0 = 0.01s$
$P_1 = 10^{-6}$ s
$A_1 = 1 m/s^2$
e = 0.6
$\omega = 2.44$ rad.

Fit parameters (curve):
$P_0 = 0.01s$
$P_1 = 9.85^{-7}$ s
$A_1 = 0.985\ m/s^2$
e = 0.614
$\omega = 2.444$ rad

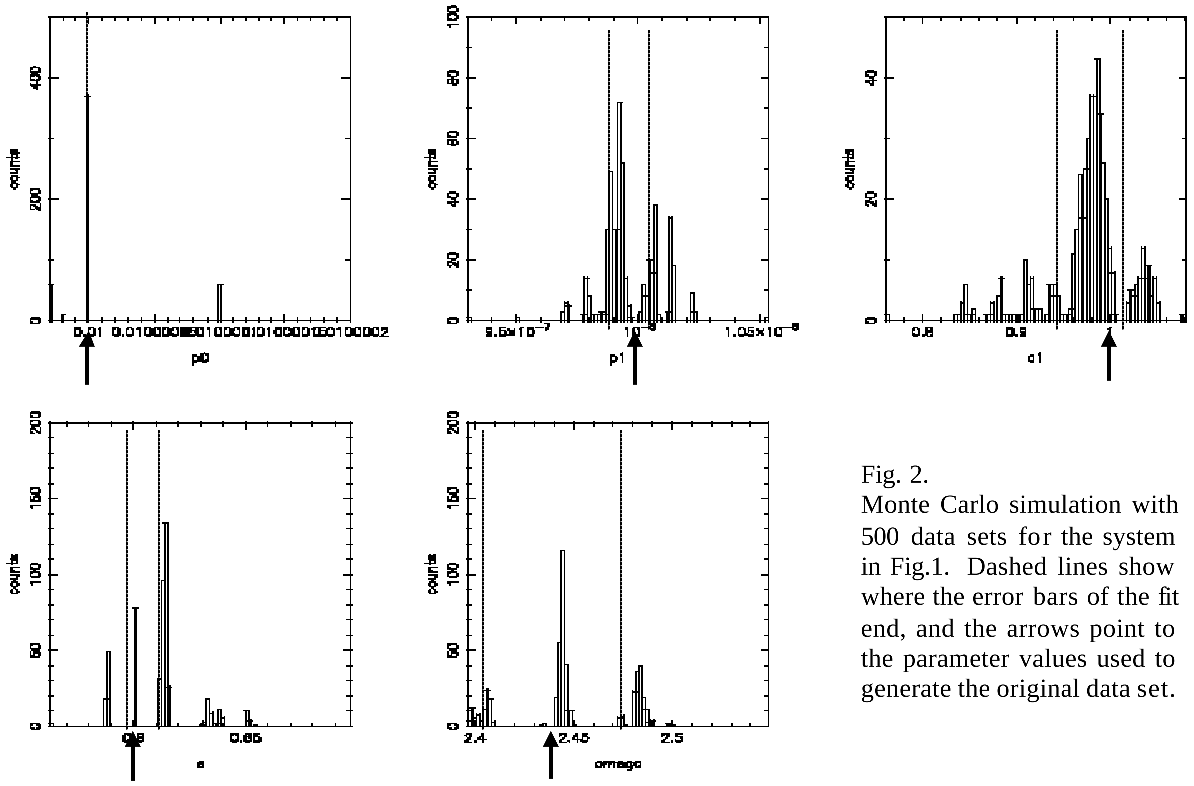

Fig. 2.
Monte Carlo simulation with 500 data sets for the system in Fig.1. Dashed lines show where the error bars of the fit end, and the arrows point to the parameter values used to generate the original data set.

References:

[1] Freire P. C., Kramer M., Lyne A. G. (2001) MNRAS **322**, 885-890
[2] Roy A. E. (1988) Orbital Motion, IoP Publishing