Relativity: An Introduction to Space, Time, and Gravity

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Relativity: What’s it all about??
[This is a set of note I prepared for the “Focus for Teens” program at Cornell in June 2002. The notes were hastily prepared, so could use some revising. If you have any comments on the notes, please feel free to email me: favata@astro.cornell.edu ]

In the pages that follow, we’ll discuss some of the following predictions of Einstein’s crazy theories:

Relativity replaces our ideas about space and time with something new: spacetime!

We will be forced to think about measurements and motion more carefully. We will find that rulers and clocks take on weird properties when they move.

Our notion of simultaneity becomes relative: just because I think two events happen at the same time, doesn’t mean that everyone agrees.

Mass and energy are no longer separate things.

Gravity is no longer a force! Instead, it arises from the curvature of space and time.

The prediction of black holes, regions where space and time get so warped, they seal themselves off from the rest of the universe.

Black holes really exist! Astronomy provides evidence for these bizarre objects.

The prediction of gravitational waves—ripples in the curvature of spacetime that propagate throughout the universe

The possibility of wormholes and time travel.

Einstein’s theory was developed in two parts: the first in 1905 is special relativity and deals with the properties of comparing measurements between two people who are moving at different speeds. The second part, called general relativity, was completed in 1915. It provides a new way to understand gravity.
Special Relativity:

Special relativity is concerned about how our concepts of length and time take on different meanings when two observers move at different speeds relative to each other.

It is based on two postulates:

1) Someone moving in a straight line at the same speed cannot tell that they are moving.

2) Everyone measures the same value for the speed for light.

[We label the speed of light with the symbol “c”. Its value is $2.9979 \times 10^8$ meters/sec = 183,300 miles/sec = 669,600,000 miles/hour = 7.5 times around the earth in 1 second.]

Our everyday experiences seem to contradict these postulates:

Suppose you are on a train, moving to the right at speed $V$. You throw a baseball on the train at speed $V_{\text{ball}}$ to your friend at the other end of the train car. Since the train moves perfectly smoothly and in a straight line, neither of you are able to determine that you are on a train by using the properties of how baseballs behave when thrown.

Your other friend is on the ground, watching the train go by. At what speed does she see the ball move? [she sees it travel with speed $V + V_{\text{ball}}$]

According to Einstein’s 2nd postulate above, does light behave like the baseball? [the 2nd postulate says NO! Your friend on the train sees the light move with speed $c$, and your friend on the ground sees it move with speed $c$ too!]

Precise physical experiments have shown that Einstein’s postulates are correct, while our everyday experience is wrong. We don’t notice the difference because we always move so much slower than light.
**Time Dilation:**

From the postulates of special relativity follow surprising consequences.

We measure time by clocks that we each carry with us. These clocks might be wristwatches, atomic clocks, your heartbeat, a regular chemical process in your cells, etc.—any perfectly periodic event from which you gauge a notion of time.

Consider a very simple type of clock, made by bouncing a light off a mirror. Take your mirror and flashlight into your spaceship and separate them by a distance $D$. Since you know the speed of light in your spaceship, the time for one “tick” of your clock [i.e. the time for the light to bounce off the mirror and come back] is $2D/c$. Suppose the clock is moving along with your spaceship at a very high speed $V$. How long does one of your clock “ticks” appear to someone at rest watching your ship go by. From the diagram above, you can see that an observer on the ground sees the light travel on a longer path. But since Einstein’s 2nd postulate says that the speed of light is the same in both frames, this means that the clock tick must be longer as viewed by someone at rest. [From the diagram above and using the Pythagorean theorem, you should be able to figure out how long one tick of the spaceship clock seems to last as measured by an identical clock at rest.]

This is called **time dilation**. We see moving clocks run slower than the clocks that we ourselves carry. What seems even stranger is that in your spaceship you find that you are at rest and that the observer on the ground is moving with speed $V$ in the opposite direction [to the left]. You see that your clock is running like normal, but the observer’s clock looks to be running slow!!
Traveling to the future:

In principle, one could use the phenomenon of time dilation to travel into the future [but not to the past]!

Get in your spaceship. Travel away from earth at a very high speed. Stop, turn around, and head back, again also at high speed. When you return to earth, everyone will have aged, but you stayed young! Suppose you wanted to go 100 years into the future. To get there in 1 year of your time, you would have to travel at 0.99995 times the speed of light. Unfortunately, most of your friends would probably be dead when you got back and your clothes would be seriously out of fashion.

But shouldn’t the person in the spaceship think that the earth was just moving away from him, so that when he returns he finds that everyone is younger? NO! The fact that the spaceship has to stop, turn around and accelerate to high speed means that the pilot knows that he is moving—the symmetry in the situation is broken since the people on earth did not feel any acceleration.

We don’t notice this effect normally because we always travel at speeds that are much smaller than light speed. However, the effect is real! An experiment was done in the 1970’s where a very accurate atomic clock was taken on an airplane that traveled around the world. When the clock was compared with one that stayed on earth, they were found to disagree by the amount that Einstein’s theory predicts.
Length Contraction

Special relativity also predicts some other bizarre effects. Just as moving clocks run slower, the lengths of objects appear to be shrunk along the direction of motion.

Consider the racecar above. With the car at rest, you and the driver both measure the length of the car to be the same, say 10 feet. But as the car whizzes past you, the driver still thinks his car is 10 feet, but you measure that it is smaller [but only in the direction of its motion. The height and width of the car are unchanged]. The apparent length of the car as seen by you has contracted!

The effect of length contraction and time dilation are very small for things moving at ordinary velocities:

Consider an F-16 moving at its maximum velocity of 1,345mph [600 m/s]. For the clock on the F-16 to differ from one on the ground by only 1 second, the F-16 would have to fly at its maximum speed for 15,833 years. At this speed the length of the plane appears to contract by only 0.00000003 mm.

The effect of special relativity are indeed quite small for even the fastest objects on earth.
Lack of Simultaneity

One of the strange consequences of special relativity is that events that are simultaneous to one observer are not simultaneous to another observer.

Consider the diagram below:

A train car moving with speed $V$ is hit by two bolts of lightening. According to observers on the ground, both ends of the train car are hit by lightening simultaneously [The observers at A and B have to correct for the time it takes the light to reach them; but after they do, they agree with the observer at C, who does not need to correct since he is in the middle.] However, because all observers measure the same speed of light, the observer at the center of the train [at $C'$] thinks that the front of the train was hit first. [Remember that the observer at $C'$ does not know that the train is moving and since he is in the middle of the train, he doesn’t have to correct for the light travel time]. Think about this for a minute or two if don’t get it.
The famous equation: \( E = mc^2 \)

Perhaps the most well known equation in science, it expresses the equivalence of mass and energy. Matter and energy are really the same thing but in different forms, and it is possible to convert one form into the other.

Because the speed of light is so large, even a very small amount of matter could provide a huge amount of energy. For example, 1 gram of material, if converted completely into energy, could power a 100-Watt light bulb for 28,500 years.

Einstein’s famous relation explains how nuclear weapons get their energy. An atomic bomb operates by the process called nuclear fission:

A neutron breaks up a large atom like Uranium into smaller fragments, plus more free neutrons. These neutrons break up more Uranium atoms and the process repeats, causing a chain reaction. For a given reaction, the sum of the masses of the fragments is less than the mass of the original Uranium atom. This “missing mass” has been converted into energy according to Einstein’s formula.
General Relativity: A theory of Gravity:

Before Einstein, gravity was described by Newton as a force that acts between objects with mass. This force $F_{\text{grav}}$ is proportional to the product of the masses $M_1M_2$ and inversely proportional to the square of the separation $r$ of the masses:

$$F_{\text{grav}} = \frac{GM_1M_2}{r^2}$$

Einstein realized that this force law did not mesh well with his relativity principles. For example, Newton’s gravitational law predicts “action-at-a-distance”: changes in the gravitational force are propagated over great distances instantaneously. Relativity, however, demands that nothing travel faster than light speed—this includes the gravitational interaction. Also, what is the “distance” $r$ in the equation of above? Is it measured by someone at rest relative to one of the masses, or to someone moving past the masses at high speed in a spaceship? Newton’s theory has an ambiguity in the meaning of this distance: it depends on who is measuring it!

At the time Einstein formulated his theory, Newton’s theory of gravity described the universe amazingly well. There was no reason to doubt its validity. However, because of his boldness and self-confidence, Einstein declared that, despite any compelling experimental evidence to the contrary, Newtonian gravity must be incorrect as it did not conform to his relativity principles—which at the time were themselves unproven.

Out of nearly pure thought and a desire for mathematical elegance, Einstein proposed that gravity was not a force of interaction between bodies with mass. Rather, gravity was the result of living in a 4-dimensional world in which space and time where “curved”. Matter and all forms of energy are what produce this curvature.
Curvature

We are already familiar with the properties of a curved space—we live on one. The surface of the earth is essentially a sphere [if you ignore the mountains and other bumps and dips on the surface].

On a small piece of this surface, it appears flat—lines that are initially parallel never cross if we extend them. Lines, curves, squares, circles, etc. follow the ordinary rules of Euclidian geometry that we learn about in school. However, the surface of the earth only appears flat to us because the earth’s radius is so much larger than the typical distances that we travel on an ordinary day. If you could measure lines and angles over very large distances, you would find that they don’t follow the ordinary rules. For example, lines that are parallel at the equator, if extended, would later intersect [as in the diagram above]. Other rules would also be different: the sum of the angles of a triangle would not add up to 180 degrees. The circumference of a circle would not be $2\pi$ times the radius.

Einstein suggested that just as a sphere is a curved 2-dimensional surface [2-dimensions since you only need 2 numbers or coordinates (latitude & longitude) to locate any point on the surface], the world we live in is actually a 4-dimensional curved surface [4-dimensions since we need to specify 3 position coordinates and one for time to say when and where something (an event) happens].

But how do we visualize something like this?? Since we already perceive space and time as separate things [even though they are not as special relativity has shown us], it is easiest to describe the two separately: a curved 3-dimensional space and a curved time.
Curved Space:

In order to imagine our curved 3-dimensional space, let us first imagine a race of 2-dimensional beings that live in a 2-dimensional space:

A region of this 2-d space is curved, but far away from this region it is flat. To picture the curved region, we draw the picture above as if it were embedded in a space with 3-dimensions [the 3rd dimension is the direction that the curved part “dips” into]. But the people who live in this 2-d world cannot perceive this 3rd dimension. However, by performing measurements they can determine that their space is in fact curved. Out in the flat region of their world, they find that parallel lines stay parallel and triangles and circles obey the ordinary rules of Euclidian geometry. But in the curved part of their world, they find that parallel lines may intersect or diverge, and circles and triangles have different properties. From these measurements they deduce the curvature of their 2-dimensional space.

In the same way, we 3-dimensional beings cannot perceive the “extra”-dimension that our space curves “into”. But by making measurements, we can determine that our 3-dimensional space is curved.
Einstein’s theory of gravity is called general relativity [“general” because it encompasses the “special” theory of relativity which doesn’t include gravity]. In this theory, matter and energy [and even the motion of matter and energy] are what cause space and time to curve. This is analogous to the curvature in a stretched sheet or a piece of rubber that is supporting a bowling ball or a heavy rock.

In our real 3-d world we would be able to detect the curvature of space around a heavy object by performing the following measurements: Take a tape measure and walk around the rock, recording the circumference of your circular path. From that path now walk towards the rock and measure the radius between the rock and your circular path around it. If you compare your measurements, you will find that the circumference C and the radius r would NOT be related by the familiar formula $C = 2\pi r$. You must then conclude that the space around the rock is curved, even though you could not perceive this curvature!

What about curved time? Einstein’s theory predicts that time is “curved” in the following sense: Suppose you have two clocks and a heavy rock like the one in the previous example. One clock stays far away from the rock. You then take the other clock, walk near the rock and stay there for a period of time. When you then go back to the clock that is far away, you find that the clock you carried with you is running slow.

**Time ticks a slower rate when you are near a massive object.**

This is a manifestation of the curvature of time.
Motion in curved spacetime

Now that we have some idea as to what curved spacetime means, how does this relate to gravity? We know that gravity exists because it makes things move. Apples fall to the ground because of the earth’s gravity. The earth orbits the sun because of the sun’s gravity. If it were not for this “force” the apple would stay in the tree, and the earth would sail right past the sun in a straight line. But in Einstein’s view of gravity, there really is no force acting on the object. Rather, the apple and the earth are just moving normally, as if there were no forces acting on them—but they are moving through a space and a time that are curved!

Using the rubber-sheet analogy again, we can think of gravity in the following way: a very massive object like the sun distorts the rubber sheet. The earth then moves on the rubber sheet just like a marble would. In Einstein’s view the earth is really moving on what are equivalent to straight lines—but straight lines in a curved, 4-dimensional spacetime! This path happens to appear curved when viewed with brains that can only perceive 3-dimensions. We can perhaps understand this better if we think about moving on the surface of the earth. Pick any direction and start walking. Eventually, you’ll walk all the way around the earth and come back to where you started. But you thought you were walking in a straight line the entire time! And you were! You were walking on a straight line of a 2-dimensional surface—the spherical earth.
General Relativity’s first test:  
The precession of Mercury.

I mentioned before that when Einstein came up with general relativity, Newton’s gravity could pretty much explain all the observations in the solar system. Actually, there remained a very slight peculiarity in the motion of the planet Mercury. Mercury’s orbit precesses, which means that it doesn’t trace out a closed ellipse. This ellipse seems to rotate by a small amount after each orbit [see diagram]. The amount of precession is measured by the angle by which the perihelion of the orbit shifts [the perihelia of the different orbits are labeled on the picture below]—the total amount of precession is 0.32 degrees per century. Most of this is due to interactions with the other planets in the solar system. But even after the effects of all the planets were taken into consideration, Mercury’s orbit still seemed to have a little “extra” precession—there was an unexplained residual precession of 0.012 degrees per century. General relativity was able to predict this extra precession, giving Einstein his first triumph over Newtonian gravity.
Gravitational Lensing

General relativity states that not only do massive bodies respond to gravity, all forms of energy do so, including light. Einstein predicted that when light from a distant star passes very close to a massive object like the sun, its path gets deflected by the sun’s gravity, causing the star to appear in a different position compared to when it is not near the sun [see the diagram below]. Einstein’s worldwide fame began when his prediction for the deflection angle was tested in 1919 and found to be correct, proving the validity of his theories and completing the overthrow of the Newtonian worldview.

In the pictures below, we can see dramatic evidence for the bending of light by gravity. Massive galaxies in the foreground act like a lens, distorting the light from objects far away. Many of the “arcs” seen in the left image are really distorted views of the same distant object. Similarly, the 4 blobs in the right image [called the Einstein cross] are also the same object.
Black Holes

Perhaps the most amazing prediction of general relativity is black holes, places where space and time become so distorted that nothing, not even light, can escape.

Consider the following: throw a ball straight up in the air. It comes down. Throw it up faster and it goes higher before coming back. But if you throw the ball up at a speed greater than 11 km/sec, it will never come back. This is the escape speed of the earth, and it depends on how strong gravity is at the earth’s surface. Now imagine we shrink the earth’s radius, but keep its mass constant. When the radius is shrunk to about 1 cm, light cannot escape the “earth’s” surface. Neither can anything else escape. The earth has become a black hole. The surface of a black hole is called the event horizon—it is the place beyond which you cannot see any “events”—anything that happens inside the event horizon is completely disconnected from the rest of the universe. Once you enter the event horizon, it is impossible to escape.

How does a black hole form?

A star is a big ball of gas that is powered by nuclear reactions, which generate heat and light. The gas and nuclear reactions produce an outward pressure. The force of gravity balances this outward pressure and keeps the star from expanding. In the picture below gravity is like the hand that squeezes the ball. The air in the ball exerts pressure, which resists the squeeze. Stars essentially work the same way.

Eventually the fuel that powers the nuclear reactions runs out, and the pressure in the star suddenly decreases. If the star is massive enough, the remaining pressure is too weak to fight off the squeeze of gravity and the star keeps contracting and
contracting and contracting... As the star gets smaller and smaller, its surface gravity keeps getting bigger. Eventually, the escape speed from the surface exceeds the speed of light [just like in our example w/ the earth above] and a black hole is formed. What happened to the star? Although the star has been cut off from our universe, trapped behind the event horizon, it keeps on contracting...until the star is compressed to nearly infinite density and squeezed into an unimaginably tiny space. This is called the singularity and lies at the center of the black hole.

What is a black hole made of?

Nothing! A black hole is quite literally made of “nothing”. It is made from the extreme warpage of space and time. In a sense, it is made out of gravity itself.

Journey to a Black Hole

Let’s explore some of the properties of a black hole. Get in your spaceship and journey to the nearest black hole. While you are still far from the black hole, send a robotic probe to explore the event horizon. You tell the probe to send out a blue light signal at a periodic interval [once a second]. You notice that as the probe gets closer to the horizon, the light signal is flashing slower than once per second. The color of the light is also changing, going from blue to green to red, and then out of the range of your vision. What you are seeing is a result of the “curved time” around a black hole. Since time ticks more slowly near the event horizon, the ticks come at a slower rate. The emission of the light itself is a kind of clock--caused by the periodic giggling of electrons. These electrons are giggling slower near the black hole, which causes the light to increase in wavelength. If you could detect all types of electromagnetic radiation [radio waves, x-rays, microwaves, etc], you would never see the probe pass through the event horizon. It would seem to take forever for the probe to reach it. The probe however, does not notice these strange effects. It passes through the event horizon rather quickly, where super-strong tidal forces eventually destroy it.
**Tidal forces:**

To understand what tidal forces are, consider the earth’s gravitational pull on an astronaut in orbit. This pull is stronger by his feet, weaker by his head, and slightly inward on his sides [see picture below]. To the astronaut, it feels like he is being stretched in one direction and squeezed in the perpendicular direction. The same thing happens to earth’s oceans due to the gravitational pull of the moon. These forces cause the oceans to bulge out and this creates the ocean tides—hence these forces are called tidal forces.

Near a black hole these tidal forces can become extremely strong. Near most black holes, the tidal forces would be more than your body could withstand. As you approach the event horizon you would be torn apart. As your remains fell toward the center of the hole, they would continue to be shredded into spaghetti like strips, until even your atoms would be ripped apart as they approached the singularity.
Realizing the harmful effects of tidal gravity, you decide its best to avoid the event horizon. Fortunately, there are interesting things to see outside the black hole. One thing you notice is that stars far in the distance behind the black hole have become distorted—this distortion increases as you get closer to the event horizon. This is a result of the bending of light that we discussed earlier. You decide to see what the sky looks like as you lower your ship closer to the horizon:

You see that the whole sky becomes squeezed into a small disk directly overhead [see picture]. This also is due to the extreme light bending that occurs near the horizon.
Black holes exist!

In the past few years, improvements in telescope technology have allowed us to gather strong evidence for black holes. In the 3 pictures below, we see images of the centers of distant galaxies taken with the Hubble space telescope. The images show a bright central region surrounded by a gassy disk. It is believed that a supermassive black hole, with a mass of a million to a billion suns lies at the heart of most galaxies. Matter forms a disk around these black holes [called an “accretion disk”]. This matter swirls into the black hole from the disk, and in the process it gets heated by the black hole’s strong gravity, giving rise to the light that we see.
Many of the images above also show powerful jets shooting out from the disk. It is believed that these jets are powered by extracting energy from a rapidly spinning black hole. Black holes are also thought to power objects called **quasars**. Quasars are the most distant objects in the universe, yet they are extremely bright. Only a supermassive black hole can generate this much power.

We also see examples of black holes that are much smaller [a few times the mass of the sun]. Below is an artist’s rendering of an object called an **X-ray binary**. A black hole and normal star orbit around each other, but the black hole pulls matter off its companion star. This matter forms a disk around the hole. As the matter falls in, it emits powerful x-rays that we can detect.

How do we know that these objects are black holes?

Most of our evidence is indirect: by studying the motion of gas around these objects we can determine their mass. From images of these objects we can also determine their size. When we compare this data, we find that a very large amount of mass is contained within a very small region, and we don’t know what else could be so compact except a black hole.
Gravitational Waves

Just as moving electric charges emit electromagnetic waves, Einstein’s theory predicts that moving masses will emit gravitational waves. These waves are actually ripples in the curvature of space and time that propagate throughout the universe. One can think of these waves like one thinks of ripples in a pond that propagate when you stir the waters with your finger.

The rapid motion of very massive objects, such as two black holes orbiting each other, produces gravitational waves. As the holes orbit each other, they lose energy that is given off as gravitational waves. This causes the orbit to shrink. Eventually, the black holes will collide and merge into a larger black hole, giving off a final burst of gravitational radiation.

What happens when a gravitational wave passes by?

The schematic below illustrates what happens as a gravitational wave passes through a ring of particles [the wave is moving “out of the page”]. You can think of a gravitational wave as a periodic wave of tidal distortion—the ring of particles is stretched along one axis and squeezed along the other in an oscillatory manner.
Detecting Gravitational Waves

The international physics community is currently engaged in a massive project to actually detect gravitational waves. The United States is leading this effort with the LIGO project, which stands for Laser Interferometer Gravitational wave Observatory.

The LIGO project will detect gravitational waves by monitoring the motion of two masses as a gravitational wave passes through them. The masses are actually big mirrors, and their motion is measured by bouncing laser light off the mirrors. The gravitational waves are so weak that they will only move the mirrors by 1/1000 the diameter of a proton [or 1/10trillion-th the diameter of a human hair]—an incredibly tiny amount. To measure such a small displacement, it is necessary to separate the mirrors in a big “L” shape, with each leg 2.5 miles long! [see diagram] Above is a picture of one the LIGO laboratories, located in Washington state. There is another identical laboratory in Louisiana, and together they will be able to detect the waves coming from colliding black holes, providing the first concrete proof of their existence.

Another way to detect gravitational waves is to accurately measure the vibrations of a great big metal bar. If a gravitational wave with just the right frequency passes through the bar, it will “ring” much like a bell that is struck with a hammer.
Wormholes and Time Travel:

If you’ve ever watched Star Trek: Deep Space 9, you’re probably familiar with the concept of a wormhole:

A wormhole is essentially a tunnel through “hyperspace” that connects different regions of space and time. Although the two “mouths” of the wormhole might be separated by an enormous distance, the travel time through the wormhole could be very quick. Wormholes are one way we can imagine exploring the distant universe. If you could construct a wormhole, it would also be possible to travel through time.

Do wormholes really exist?

Wormholes are perfectly valid solutions to Einstein’s equations of general relativity. However, these wormholes are found to be unstable—if anything tried to go through the wormhole [even a tiny speck of dust], the hole would immediately collapse before the object could get through.

It might be possible for a very advanced alien civilization to construct a wormhole if they could make a very strange type of matter. However, it is not yet clear if the laws of quantum mechanics allow this type of matter to exist. Furthermore, the possibility of constructing a time machine from a wormhole opens up the possibility for a variety of paradoxical situations: for example, you could go back in time and kill your grandmother, so you would never be born—but how then could you go back in time and kill your grandmother...?? Because of these strange possibilities, Stephen Hawking has conjectured that the laws of physics must somehow prevent the construction of a device that would allow you to travel backwards in time. [It’s still possible to travel forward in time though, as we discussed previously]. The bottom line is that it is very unlikely that traversable wormholes and time machines exist, but we have not yet been able to prove that this is so.
Conclusion:

Our whirlwind tour through Einstein’s outrageous universe must now come to an end. If there is one thing that you should take away with you it is this: The universe really is this crazy. Einstein’s theories are not the just the ravings of a mad genius—they make precise predictions about nature—predictions that have been tested to high accuracy in hundreds of experiments. No experiment has ever contradicted the theory of relativity.

There are places where we expect that Einstein’s theory will fail—at the birth of the universe, near the singularity of a black hole—places where the predictions of an even more bizarre theory, quantum mechanics, become important. But that is a story for another day.

References and further reading:

Kip S. Thorne, “Black Holes and Time Warps: Einstein’s Outrageous Legacy”, Norton (New York), 1994. [This the best introductory reference on this subject. Several of the figures used these notes were scanned from this book.]

Paul A. Tipler, “Physics for Scientists and Engineers”, 3rd extended edition, Worth (New York) 1991. [This is an introductory (calculus-based) college physics textbook that I like. I also borrowed figures from here.]

The web is probably the best place to start to learn more about relativity. Maybe in the future I’ll put some links here. For, now use www.google.com.

[Note, that ALL of the figures in these notes were scanned from books (mostly the ones cited above) or taken from webpages. None of them are my own. I have made no attempt to determine where I got these figures from nor do I intend to in the future. They will be used only for educational or research purposes. ]

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