Are neutron stars crushed?
Gravitomagnetic tidal fields as a mechanism for binary-induced-collapse

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Motivations

• Wilson-Mathews revised simulations still show small compression.
• Over 15 papers refuted their original claims
• A “loophole” remains:
  – all analytic papers neglected gravitomagnetic-velocity couplings
  – numerical papers assumed corotation or irrotation or used ellipsoidal stellar models
  – [some evidence for binary-induced-collapse in particle clusters; (Shapiro, Duez et al., Alvi & Liu)]

• Question of principle: is there any way to compress a NS?
How to crush a neutron star?

Gravitomagnetic tidal forces act on the star:

\[ a^\text{ext}_i = -\nabla_i \Phi^\text{ext} - \zeta^\text{ext}_i + (\nu \times B)_i \]

\[ \Phi^\text{ext} = \frac{1}{2} \mathcal{E}_{ab} x^a x^b \]

\[ \zeta^\text{ext}_i = -\frac{1}{3} \varepsilon_{ipq} B^p q x^q x^l \]

\[ B = \nabla \times \zeta^\text{ext} \]

\[ S_{ij} = \int x(\varepsilon_{i,j})_{ab} x_a \rho v_b \, d^3 x \]
How to crush a neutron star?

Gravitomagnetic tidal forces act on the star:

\[
a^\text{ext}_i = -\nabla_i \Phi^\text{ext} - \dot{\zeta}^\text{ext}_i + (\nu \times B)_i
\]

\[
\Phi^\text{ext} = \frac{1}{2} \mathcal{E}_{ab} x^a x^b
\]

\[
\zeta^\text{ext}_i = -\frac{2}{3} \epsilon_{ipq} B^p_i x^q x^l
\]

\[
B = \nabla \times \zeta^\text{ext}
\]

resulting change in central density:

\[
\frac{\delta \rho_c}{\rho_c} = c_1 \mathcal{E}_{ij}(t) \mathcal{E}^{ij}(t) + c_2 \mathcal{B}_{ij}(t) \mathcal{B}^{ij}(t)
\]

\[
O\left[\left(\frac{R}{d}\right)^6\right]\]

\[
O\left[\left(\frac{R}{d}\right)^7\right]
\]
Details I: Eulerian Perturbation expansion

start with fluid equations:

\[ \frac{\partial \rho}{\partial t} + \nabla_i (\rho v^i) = 0 \]

\[ \frac{\partial v_i}{\partial t} + (v^k \nabla_k) v_i = -\frac{\nabla_i P}{\rho} - \nabla_i \Phi + a^\text{ext}_i \]

\[ \nabla^2 \Phi = 4\pi \rho \]

\[ a^\text{ext}_i = \varepsilon \left(\frac{2}{3} \epsilon_{ijk} \dot{B}^j_i x^k x^l - 2\epsilon_{ijk} B^{kl} v^j x^l\right) \]

expand them:

\[ \rho(t, x) = \rho^{(0)} + \varepsilon \rho^{(1)} + \varepsilon^2 \rho^{(2)} + \cdots \]

\[ P(t, x) = P^{(0)} + \varepsilon P^{(1)} + \varepsilon^2 P^{(2)} + \cdots \]

\[ \Phi(t, x) = \Phi^{(0)} + \varepsilon \Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \cdots \]

\[ v(t, x) = v^{(0)} + \varepsilon v^{(1)} + \varepsilon^2 v^{(2)} + \cdots \]
Details I: Eulerian Perturbation expansion

at order $O(\varepsilon^0)$:

$$\nabla_i P^{(0)} = -\rho^{(0)} \nabla_i \Phi^{(0)}, \quad \dot{\rho}^{(0)} = 0$$

at order $O(\varepsilon^1)$:

$$\rho^{(1)} = P^{(1)} = \Phi^{(1)} = 0$$

$$\nu_i^{(1)} = -\zeta_i^\text{ext} = \frac{2}{3} \epsilon_{ijk} B^j \epsilon^k_l x^k x^l$$

This leads to an induced current quadrupole moment:

$$S_{ij} = \gamma_2 M R^4 B_{ij} \quad \text{where} \quad \gamma_2 = \frac{2}{15} \int_0^{\xi_1} \frac{\xi^6 \theta^n \, d\xi}{\xi_1^6 |\theta'(\xi_1)|}$$

is the gravitomagnetic Love number.

Analogous to the Newtonian Love number $k_2$:

$$I_{ij} = -\frac{1}{3} k_2 R^5 \mathcal{E}_{ij}$$

See poster by Hinderer & Flanagan: Monday
Details I: Eulerian Perturbation expansion

at order $O(\varepsilon^2)$ :

\[
\frac{\partial \rho^{(2)}}{\partial t} + \nabla_i [\rho^{(0)} v_i^{(2)}] = 0
\]

\[
\frac{\partial v_i^{(2)}}{\partial t} + \frac{\nabla_i P^{(2)}}{\rho^{(0)}} + \nabla_i \Phi^{(2)} + \frac{\rho^{(2)}}{\rho^{(0)}} \nabla_i \Phi^{(0)} = a_i^{\text{tot}}
\]

\[
a_i^{\text{tot}} = (v^{(1)} \times B)_i - (v^{(1)} \cdot \nabla) v_i^{(1)}
\]

Can convert this to an equation for first-order Lagrangian perturbations of a spherical star [ch. 6 of Shapiro & Teukolsky]

\[
\ddot{\xi}_i + \mathcal{L}_{ij} [\xi_j] = a_i^{\text{tot}}
\]

Solve for the fundamental radial mode to determine the change in central density.
Results I: Change in central density

\[ \frac{\delta \rho_c}{\rho_c} = -2.280 \left( \frac{M'}{M} \right)^2 \left( \frac{R}{d} \right)^6 + 1.157 \left( \frac{M}{R} \right)^2 \left( 1 + \frac{M'}{M} \right) \left( \frac{M'}{M} \right)^2 \left( \frac{R}{d} \right)^7 \]

Tidal stabilization term \hspace{1cm} \text{Gravitomagnetic compression term}

[Lai; Taniguchi & Nakamura]

Although compression increases with mass ratio, tidal disruption or ISCO is \textit{always} reached before compression dominates.

Exception: contrived configurations in which Newtonian tidal field vanishes.
Details II: Pre-existing velocity field

However, it is possible to get a net compression if there is a pre-existing velocity field.

Consider a general expansion of the stellar velocity in vector spherical harmonics:

\[ v_0(t, x) = \sum_{lm} E_{lm}^{E}(t, r) Y^{E,lm} + B_{lm}^{B}(t, r) Y^{B,lm} + R_{lm}^{R}(t, r) Y^{R,lm} \]

Suppose that in isolation, this velocity satisfies: \( \nabla \cdot (\rho v_0) = 0 \), \( v_0^2 \ll M/R \)

The only angle-averaged radial force comes from the Lorentz-like term in the external acceleration: \( v_0 \times B \)

\[ \frac{\delta \rho_c}{\rho_c} = c_1 \mathcal{E}_{ij}(t) \mathcal{E}^{ij}(t) + c_2' \mathcal{S}_{ij}(t) \mathcal{B}^{ij}(t) \]

\[ O\left(\left(\frac{R}{d}\right)^6\right) \quad O\left[\left(\frac{R}{d}\right)^{7/2}\right] \]
Results II: Change in central density

\[
\frac{\delta \rho_c}{\rho_c} = -2.280 \left( \frac{M'}{M} \right)^2 \left( \frac{R}{d} \right)^6 + 5.493 V_S \left( \frac{M}{R} \right)^{1/2} \left( \frac{M'}{M} \right) \left( 1 + \frac{M'}{M} \right)^{1/2} \left( \frac{R}{d} \right)^{7/2}
\]

Tidal stabilization term  
Gravitomagnetic compression term

[Lai; Taniguchi & Nakamura]

where we defined a characteristic velocity of the current quadrupolar motions:

\[ |S_{ij}| \approx MR^2 V_S \]
Results II: Change in central density

\[ \frac{\delta \rho_c}{\rho_c} = -2.280 \left( \frac{M'}{M} \right)^2 \left( \frac{R}{d} \right)^6 + 5.493 V_S \left( \frac{M}{R} \right)^{1/2} \left( \frac{M'}{M} \right) \left( 1 + \frac{M'}{M} \right)^{1/2} \left( \frac{R}{d} \right)^{7/2} \]

\[ V_S = 0.1 \left( \frac{M}{R} \right)^{1/2} \]

and \( \left( \frac{M'}{M} \right) = 1, 3, 5, 10 \)
Conclusions

• For non-rotating stars that are unperturbed in isolation, a compressive force does exist, but it is always smaller than the Newtonian tidal stabilization. (Except for contrived situations.)

• If the star has a pre-existing current-quadrupolar velocity component, net compression is possible. The change in central density is small, but could, in principle, cause collapse in a star very, very close to its maximum mass.

• What does this say about the Wilson-Mathews compression: probably nothing, but binary-induced compression is possible at least in principle.

• It would be worthwhile to perform binary simulations without constraints on the hydrodynamics. These effects (and r-modes too) vanish for irrotational configurations. Probably unimportant for NS/NS mergers, but might be important for NS/BH mergers.